

impurities and heat to the growth interface. The modification of fluid motion consists in the quenching of flow oscillations, and therefore, the temperature fluctuations of which they are responsible. This improves the resulting crystal quality by avoiding the oscillatory crystal growth, which may result from temperature fluctuations and which is characterised by a non-uniform distribution of dopant in the crystal (Hurle, 1966; Utech and Flemings, 1966).

The effect of a constant magnetic field on the liquid metal natural convection flows in various configurations was the subject of many studies (Series and Hurle, 1991; Garandet and Alboussière, 1999; Mebarek-Oudina and Bessaïh, 2007). Many works were interested in the Bridgman horizontal configuration in rectangular or cylindrical cavities (Benhadid and Henry, 1996; 1997). Various intensities and orientations of the magnetic field were considered. Touihri *et al.* (1999) investigated a three-dimensional numerical study in a cylindrical submitted to an magnetic field. They obtained the stability diagrams. Gelfgat *et al.* (2001) have been carried out a study of the effect of a vertical magnetic field on the axisymmetric flow instability in a cylindrical enclosure. The cylinder is heated by bottom and a temperature parabolic profile is imposed on the side wall. Stability diagrams were obtained for a Prandtl number ( $Pr = 0.015$ ) and for various aspect ratios.

The present work investigates numerically the determination of thermal instabilities which are created in a

cylindrical enclosure having an aspect ratio equal to 2 filled with a liquid metal, and subjected to an axial magnetic field  $B_0$  and axial temperature gradient. We determine the critical value of the critical Grashof number  $Gr_c$  for each value of the Hartman number.

## 2. Geometry and mathematical model

The geometry of the flow field analysed in this study is illustrated in Figure 1. A liquid metal with a density  $\rho$ , a kinematics viscosity  $\nu$  and an electrical conductivity  $\sigma$ , fills a cylinder of radius  $r_c$  and height  $H$  is submitted to an axial magnetic field  $B_0$ . The bottom wall is kept at a local hot temperature  $T_H$ , the top wall is maintained at a local cold temperature  $T_C$  ( $T_C < T_H$ ), and the sidewall is adiabatic. The flow is assumed incompressible and axially symmetric. The application of an axial magnetic field is known as a good candidate for stabilising the flow: this should prevent the appearance of azimuthal instabilities. Neglecting the induced magnetic field, the dissipation and Joule heating, and the Bousinesq approximation is valid (Bejan, 1995); and using  $r_c^2/\nu$ ,  $r_c$ ,  $\nu/r_c$ ,  $\rho(\nu/r_c)^2$  and  $(T_H - T_C)$  as typical scales for time, lengths, velocities, pressure, and temperature, respectively, the dimensionless governing equations for the conservation of mass, momentum and energy together with appropriate boundary conditions in the cylindrical coordinates system  $(r; Z)$ , are written as follows:

$$\frac{1}{R} \frac{\partial}{\partial R} (R.U) + \frac{\partial V}{\partial Z} = 0 \quad (1)$$

$$\frac{\partial U}{\partial \tau} + \frac{1}{R} \frac{\partial}{\partial R} (R.U^2) + \frac{\partial}{\partial Z} (V.U) = - \frac{\partial P}{\partial R} + \left( \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) + \frac{\partial^2 U}{\partial Z^2} - \frac{U}{R^2} \right) + F_R \quad (2)$$

$$\frac{\partial V}{\partial \tau} + \frac{1}{R} \frac{\partial}{\partial R} (R.U.V) + \frac{\partial}{\partial Z} (V^2) = - \frac{\partial P}{\partial Z} + \left( \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) + \frac{\partial^2 U}{\partial Z^2} \right) + Gr.\theta + F_Z \quad (3)$$

$$\frac{\partial \theta}{\partial \tau} + \frac{1}{R} \frac{\partial}{\partial R} (R.U.\theta) + \frac{\partial}{\partial Z} (V.\theta) = \frac{1}{Pr} \left( \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \theta}{\partial R} \right) + \frac{\partial^2 \theta}{\partial Z^2} \right) \quad (4)$$

Where  $F_R$  and  $F_Z$  represent the components of Lorentz force in directions  $r$  and  $z$ , respectively, which

have been obtained by the using the equation (Bessaïh *et al.*, 1999):  $F = J_{\times B}$ , where  $J$  and  $B$  are the electric current