

and magnetic field vectors, respectively. The expressions of these components are:

$$F_R = -Ha^2 \cdot U \quad (5a)$$

$$F_z = 0 \quad (5b)$$

The above equations have been solved subject to the following initial and boundary conditions:

- At  $\tau = 0$ ,  $U = V = \theta = 0$
- For  $\tau > 0$ ,
  - At  $R = 0$ :  $U = \partial V / \partial R = 0$ ,  $\partial \theta / \partial R = 0$ ; axis of symmetry
  - At  $R = 1$ :  $U = V = 0$ ,  $\partial \theta / \partial R = 0$ ; lateral wall
  - At  $Z = 0$ :  $U = V = 0$ ,  $\theta = 1$ ; bottom wall
  - At  $Z = H/r_c$ :  $U = V = 0$ ,  $\theta = 0$ ; top wall

### 3. Numerical method

The governing equations were solved using a finite volume method (Patankar, 1980). Scalar quantities ( $P$ ,  $\theta$ ) are stored in the centre of these volumes, whereas the vectorial quantities ( $U$ ,  $V$ ) are stored on the faces of each volume. For the discretisation of spatial terms, a second-order central difference scheme was used for the diffusion and convection parts of the mathematical model, Eqs.1-4, and the SIMPLER algorithm (Patankar, 1980) was used to determine the pressure from continuity equation. Calculations were carried out on a PC with CPU 2.8 GHz, thus the average computing time for a typical case was approximately of 12 hours.

## 4. Results and discussion

### 4.1. Grid independency

The increments  $\Delta r$  and  $\Delta z$  of the grid are not regular. They were chosen according to geometric progressions of ratio equal to 1.05, which permitted grid refinement near the walls, i.e. in the Hartmann layer where large velocity and temperature gradients exist, thus requiring a larger number of nodes in order to resolve the specific characteristics of the MHD flow, also in order to reduce numerical errors.

In order to examine the effect of the grid on the numerical solution, a number of grid sizes have been investigated for grid independence:  $32 \times 62$ ,  $52 \times 102$  and  $72$

$\times 142$  nodes. By increasing the grid size from a  $52 \times 102$  to  $72 \times 142$  nodes, less than 5 % change in computed values was observed (Table 1).

Table 1

Effect of the grid on the values of  $\overline{Nu}$ ,  $U_{max}$ ,  $V_{max}$ ,  $|\Psi_{max}|$ , and  $|\Psi_{min}|$ , for  $Gr = 10^5$ ,  $Pr = 0.015$  and  $A = 2$ .

	32×62 nodes	52×102 nodes	72×142 nodes
$\overline{Nu}$	5.000	5.000	5.000
$U_{max}$	6.119	6.081	6.517
$V_{max}$	18.513	18.403	19.281
$ \Psi_{max} $	$6.72 \times 10^{-5}$	$3.13 \times 10^{-5}$	$2.87 \times 10^{-5}$
$ \Psi_{min} $	-1.156	-1.161	-1.223

Therefore, the grid captures best the dynamic and thermal boundary layers near the walls. The later was used in all calculations presented in this paper after performing grid independency tests.

### 4.2. Validation of the code

Firstly, the code used in this work was validated with the experimental data obtained by Michelson (1986) who used the LDA technique (Laser Doppler Anemometer) to determine velocity distributions in a cylindrical cavity with an aspect ratio  $H/r_c = 1.0$ , where the fluid rotates at a Reynolds number,  $Re = \Omega \cdot r_c^2 / \nu = 1800$ , where  $\Omega$  is the angular velocity of the top disk. Figures 2a-b show the radial and azimuthal velocities distributions at constant radius  $R = 0.7$ . The computed values can be seen to be in excellent agreement with measurements over the whole flow field.

Secondly, the comparison was made with the numerical results of Iwatsu (2004), which has used the finite differences method in order to discretise the mathematical model. The geometry considered here concerns a cylinder having an aspect ratio  $A = 1$  and fills with a fluid whose Prandtl number equal to 1, and having an aspect ratio  $A = 1$ . It took into account the heat transfer between the cooled bottom wall and the heated top rotating disk. With this intention, we have chosen several mesh:  $42 \times 42$ ,  $82 \times 82$  and  $162 \times 162$  nodes. We can conclude by examining the Table 2, which shows the flow parameters of our numerical results. A good agreement between our numerical simulations and those of Iwatsu (2004) is obtained.