

the carbides is neglected. Then the growth of austenite, which is carbon diffusion controlled, can be described by Fick's second law applied with correct initial and boundary conditions.

If γ , α and θ are used to denote austenite, ferrite and cementite respectively, as shown in Figure 4, the carbon concentration C_r in austenite is a function of time t and distance from the centre of the cementite particle r (Brooks, 1992; Larsson and Karlsson, 1975):

$$\frac{\partial C_\gamma}{\partial t} = \frac{D_\gamma}{r^n} \frac{\partial}{\partial r} \left[r^n \frac{\partial C_\gamma}{\partial r} \right] \quad (1)$$

where D_r is the carbon diffusion coefficient in austenite which is a function of carbon concentration of C_r and temperature T during austenitization process. The evaluation of carbon diffusion coefficient has been discussed elsewhere (Bhadeshia, 1981; Siller and McLellan, 1969; Siller and McLellan, 1970; Larsson and Karlsson, 1975; Jacot and Rappaz, 1997; Jacot and Rappaz, 1999; Jacot et al., 1998; Karlsson and Larsson, 1975; Wert, 1950; McLellan and Dunn, 1969; Capdevila et al., 2001; Smith, 1946; Wells et al., 1950; Smith, 1964; Smithells, 1967). Usually, it is assumed to be concentration dependent for an isothermal austenitization process (Atkinson and Akbay, 1996; Larsson and Karlsson, 1975; Larsson and Karlsson, 1975; Smith, 1946; Wells et al., 1950; Smith, 1964; Smithells, 1967) and temperature and concentration dependent for a thermal austenitization process (Jacot and Rappaz, 1997; Jacot and Rappaz, 1999; Jacot et al., 1998; Wert, 1950; McLellan and Dunn, 1969; Capdevila et al., 2001). n is an integer related to the geometry of an austenitization model; $n = 0$ for planar geometry, $n = 1$ for cylindrical geometry and $n = 2$ for spherical geometry.

At the cementite/austenite interface:

$$(C_\theta - C_\gamma^{\theta}) \frac{dr_{\gamma\theta}}{dt} = D_\gamma \left(\frac{\partial C_\gamma}{\partial r} \right)_{r=r_{\gamma\theta}^+} \quad (2)$$

At the austenite/ferrite interface:

$$(C_\gamma^{\alpha} - C_\alpha) \frac{dr_{\gamma\alpha}}{dt} = -D_\gamma \left(\frac{\partial C_\gamma}{\partial r} \right)_{r=r_{\gamma\alpha}^-} \quad (3)$$

where $r_{\gamma\theta}$ and $r_{\gamma\alpha}$ are the instantaneous positions of the θ/γ and γ/α interfaces respectively (Fig. 4.). The superscripts

“+” and “-” indicate the directions of $\partial C_\gamma / \partial r$ to the austenite region at $r_{\gamma\theta}$ and $r_{\gamma\alpha}$. C_γ^{θ} and C_γ^{α} are carbon content in austenite at the θ/γ and γ/α interfaces, which can be determined by the numerical minimization of Gibbs energy (Akabay et al., 1994). The initial boundary conditions ($t=0$) are given as $r_{\gamma\theta}|_{t=0} = r_{\gamma\alpha}|_{t=0} = r_0$; $C = C_\theta$ for $0 \leq r \leq r_0$ and $C = C_\alpha$ for $r_0 \leq r$ where r_0 is the initial radius of the cementite particle. Local equilibrium at the θ/γ and γ/α interfaces are $C_\gamma|_{r=r_{\gamma\theta}^+} = C_\gamma^{\theta}$ and $C_\gamma|_{r=r_{\gamma\alpha}^-} = C_\gamma^{\alpha}$.

The carbon concentration decreases in the growing austenite region, which is from C_γ^{θ} to C_γ^{α} , follows the leverage rules in the Fe-C diagram as shown in Figure 1 a). The value of carbon concentration drops at the θ/γ and γ/α interfaces depends on the austenitization temperature. Numerical and analytical solutions of the above equations for planar and spherical geometry can refer to (Akabay et al., 1994; Judd and Paxton, 1968; Atkinson and Akbay, 1996). It should be noted that, there must be a carbon concentration gradient in the ferrite region as well due to the carbon diffusion at γ/α interfaces, as shown in Figure 4 c). This carbon concentration gradient in ferrite is usually neglected due to the low solubility and diffusivity of carbon in ferrite (Akabay et al., 1994; Judd and Paxton, 1968; Reed et al., 1998; Atkinson and Akbay, 1996). The assumption is fairly acceptable for steels with a relatively high carbon content, but may be not accurate enough for a low carbon micro-alloyed steel, which contains mainly ferrite. The carbon diffusion in ferrite plays a much more important role in this case. However, little effort has been made to investigate the carbon concentration gradient in the ferrite phase during austenitization of steels and no material model is available for this purpose.

It may happen that the cementite in the parent material has not been fully dissolved even though the ferrite has been fully austenitized (Fig. 5). This is shown in Figure 4, when $r_{\gamma\alpha}$ reaches the cell boundary a , but $r_{\gamma\theta}$ does not reach the symmetry axis ($r=0$) simultaneously. In such cases, extra time is required for the dissolution of the retained cementite and austenite to homogenize. Compared with the nucleation and grain growth of austenite, the homogenization process takes longer. Fick's second law can be applied to model austenite homogenization which is carbon diffusion controlled. In Karlsson and Larsson's model (Karlsson and Larsson, 1975; Karlsson and Larsson,