

1975), homogenization of austenite can be described by the same set of equations as that used to model austenite grain growth. The only difference is the application of different boundary and interface conditions.

### 3.2. Austenitization between $\alpha/\gamma$ or $\theta/\gamma$ phases

For a general case, it is assumed in the model that the ranges of existence of the two phases,  $\alpha$  and  $\gamma$ , or  $\theta$  and  $\gamma$  phases, are limited and separate. The diffusion of carbon in ferrite and cementite is ignored. The concentration at the interface of both phases is considered constant (the transformation at the interface is assumed infinitely rapid) and no mass transport across the cell boundary occurs, and molar volume is the same for all  $\gamma$ ,  $\theta$  and  $\alpha$  phases. If using  $\delta$  represents a single  $\alpha$  phase or  $\theta$  phase, then in  $\gamma$  phase:

$$\frac{\partial C_\gamma}{\partial t} = \frac{1}{r^n} \frac{\partial}{\partial r} \left[ r^n D_\gamma (C) \frac{\partial C_\gamma}{\partial r} \right] \quad 0 < r < y(t) \quad (4)$$

where  $y(t)$  is interface coordinate along  $r$  direction (Fig. 7). The interface mass balance is

$$(C_\delta^{\gamma\delta} - C_\gamma^{\gamma\delta}) \frac{dy}{dt} = D_\gamma \left( \frac{\partial C_\gamma}{\partial r} \right)_{r=y^-} - D_\delta \left( \frac{\partial C_\delta}{\partial r} \right)_{r=y^+};$$

The boundary conditions are

$$\left. \frac{\partial C_\gamma}{\partial r} \right|_{r=0,a} = 0, C(y(t)^-, t) = C_\gamma^{\gamma\delta} \text{ and } C(y(t)^+, t) = C_\delta^{\gamma\delta}$$

The initial condition is

$$C(r, 0) \Big|_{0 \leq r < y(0)=r_0} = C_\gamma \text{ and } C(r, 0) \Big|_{y(0)=r_0 < r \leq a} = C_\delta,$$

where  $a$  is the cell boundary,  $r_0$  is the initial interface position of the  $\gamma$  and  $\delta$  phases. Numerical solution of Eq. (4) is given in (Karlsson and Larsson, 1975).

After the interface reaches the cell boundary ( $x=a$ ), the remaining diffusion is limited to the  $\gamma$  phase and a homogeneous  $\gamma$  phase finally results. The initial carbon concentration profiles and interface concentrations are taken from the Fe-C phase diagram, mean composition, cell size, diffusion coefficients and node spacing are manually determined.

Figure 7 illustrates an austenitization process when applying the model to a parent material composed of two phases: pearlite and ferrite (Fig. 7 a).

From the beginning, in a pearlite grain, there are many  $\alpha$  and  $\theta$  phases interfaces. The diffusion of carbon in the lamellar pearlite is on a nano scale, based on the zoomed out microstructure in Figure 7 a), which is exactly

the same planar model as that shown in Figure 4. Austenite can nucleate at the interfaces simultaneously and the whole pearlite grain can be austenitized instantaneously when reaching the austenitization temperature (Karlsson and Metallkd, 1972). Therefore, the forming and growing rates of austenite in a lamellar pearlite structure is much faster compared with that in ferrite. Then the initial phase can be taken as ferrite and the austenite formed from pearlite with an initial carbon concentration of 0.8% when  $t=0$  (Karlsson and Larsson, 1975) (Fig. 7), neglecting austenitization of pearlite. In this case, austenitization can be treated as austenite grain growth from the initial austenite when  $t=0$  with a carbon concentration of 0.8% into ferrite (Fig. 7 b).

Figure 8 is the calculated carbon concentration profile during austenitization, in this case under an isothermal diffusion condition, with the cylindrical model. Initial austenite grain size is set to be  $4 \mu\text{m}$ , which can also represent the original pearlite grain size in the parent material. The grain boundary to the center of symmetry is  $7.06 \mu\text{m}$ . The value of carbon diffusion coefficient is measured from the experimental curve of carbon concentration dependence on the diffusion coefficient when the austenitizing temperature is  $758^\circ\text{C}$  (Wells et al., 1950; Smith, 1964; Smithells, 1967). Solving Eq. (4) gives the carbon distribution profile in each phase and time dependent interface position changes. It can be seen that the carbon concentration in austenite decreases with the increasing time, and the  $\gamma/\alpha$  interface moves to the right hand side, which represents austenite growth and ferrite shrinkage. Because the position of the  $\gamma/\alpha$  interface is determined, the volume fraction of austenite as a function of time can also be calculated.

### 3.3. Kinetic models

A global optimization technique to identify the overall kinetics of austenitization was developed by Tszeng and Gshi (2004). The kinetic model employed for the nucleation and growth of austenite, Eq. (6), is developed from the Avrami equation, Eq. (5):

$$f = 1 - \exp(-A_1 t^{A_2}) \quad (5)$$

where  $f$  is the transformed volume fraction of austenite, and  $A_1, A_2$  are temperature dependent parameters.