

techniques. The objective function for this procedure is taken to be the difference in the least-squares sense between the computed and the measured natural frequencies of the system. The design variables are defined in terms of some undetermined parameters on boundary and the sensitivities with respect to these parameters are directly obtained from the boundary element formulation by using the differentiation approach. The conjugate gradient algorithm for unconstrained optimization is adopted for minimizing the objective function.

2. Fundamental equations

Consider a thin plate resting on an elastic foundation and occupying the region Ω bounded by the boundary

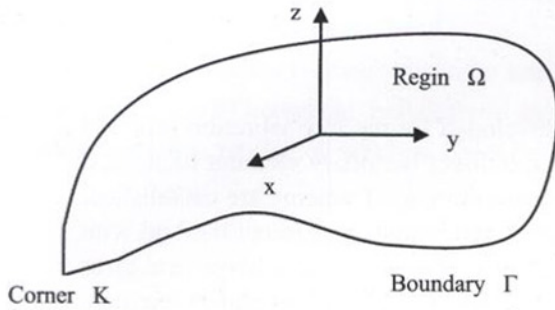


Fig.1. Plate resting on an elastic foundation.

curves Γ (Fig. 1).

The governing differential equation dealing with the free vibration can be expressed as

$$D \nabla^4 W + (K - \rho \omega^2) W = 0 \tag{1}$$

where D is the bending stiffness of the plate, K is the foundation modulus, ρ is a mass density of the plate, and ω is the frequency parameter of the system.

Moreover, the deflection W must satisfy the boundary conditions and the corner conditions. In order to simulate the various boundary restraints, the following generalized boundary conditions and corner conditions are employed

$$\left. \begin{aligned} \alpha_1 W + \alpha_2 Q &= \alpha_3 \\ \beta_1 \Theta + \beta_2 M &= \beta_3 \end{aligned} \right\} \text{ on } \Gamma \tag{2}$$

$$C_{1k} W^{(k)} + C_{2k} R^{(k)} = C_{3k} \text{ at the corner point } k \tag{3}$$

In expressions (2) and (3), α_i , β_i and C_{ik} ($i=1, 2, 3$) stand for undetermined parameters, respectively defined on the boundary Γ and at the corner point k . W , Θ , M and Q respectively are the deflection, rotation, bending moment and Kirchhoff equivalent shear force along the plate boundary, $W^{(k)}$ and $R^{(k)}$ are the corner deflections and corner concentrated forces, which can be expressed as :

$$\left. \begin{aligned} \Theta &= \frac{\partial W}{\partial n} \\ M &= -D \left[\mu \nabla^2 W + (1 - \mu) \frac{\partial^2 W}{\partial n^2} \right] \\ Q &= -D \left\{ \frac{\partial}{\partial n} \left[\nabla^2 W + (1 - \mu) \frac{\partial^2 W}{\partial s^2} \right] - (1 - \mu) \frac{\partial}{\partial s} \left(\frac{1}{\rho} \frac{\partial W}{\partial s} \right) \right\} \\ R^{(k)} &= -(1 - \mu) D \left(\frac{\partial^2 W}{\partial n \partial s} - \frac{1}{\rho} \frac{\partial W}{\partial s} \right)^{[k]} \end{aligned} \right\} \tag{4}$$

where n and s respectively stand for the outward normal and tangent of the boundary Γ ; $(\dots)^{[k]}$ means the discontinuous jump at the point k .

The boundary conditions (2) and the corner conditions (3) are more suitable for the practical problem, from which many kinds of conventional boundary conditions, including the mixed boundary conditions, can be derived.

3. Boundary element formulation

We define the fundamental solution of the equation (1) by

$$D \nabla^4 W^* + (K - \rho \omega^2) W^* = \delta(r) \tag{5}$$

The fundamental solution of the problem can be derived by Hankel transform (Puttonen and Varpasuo,