1986). By using the Kelvin functions of the second kind, the fundamental solution can be expressed in the following form

$$W^*(r) = -\frac{1}{2\pi D}\sqrt{\frac{D}{K-\rho\omega^2}}\,Kei\left(\sqrt[4]{\frac{K-\rho\omega^2}{D}}\,r\right) \tag{6}$$

The boundary integral equation for the direct method can be derived by using the general reciprocal theorem(Bezine, 1988), which can be expressed in the following form

$$C\,W(P) = \int_\Gamma \left[W^*Q - \Theta^* M + M^*\Theta - Q^*W\right]d\Gamma + \sum_k \left[\left(W^*\right)^{(k)} R^{(k)} - \left(R^*\right)^{(k)} W^{(k)}\right] \tag{7}$$

where the value of a coefficient C depends on the position of the point P. When $P \rightarrow \Gamma$, C is the inner angle between the tangents.

From equation (7) all other variables can be computed when the values of W, Θ, M, Q and $R^{(k)}$ are known. For the solution of these unknown variables, two boundary integral equations are needed at every boundary point. One of these can be obtained by $P \rightarrow \Gamma$ in equation (7) and the other can be obtained by differentiating equation (7). By using the notation $W_1^* = W^*$ and $W_2^* = \partial W^* / \partial n$, these two boundary integral equations can be written as follows :

$$CW(P) = \int_\Gamma \left[W_i^*Q - \Theta_i^* M + M_i^*\Theta - Q_i^*W\right]d\Gamma + \sum_k \left[(W_i^*)^{(k)} R^{(k)} - (R_i^*)^{(k)} W^{(k)}\right] \; (i=1,2) \tag{8}$$

These boundary integral equations (8), together with the boundary conditions (2) and corner conditions (3) constitute a set of simultaneous equations, which can be solved to yield the solution of the problem.

Now we discretize the boundary Γ into boundary elements and require that all of the boundary corner are contained in the nodes. Inside the element a boundary variable is defined as a linear function of its nodal values. By using the standard boundary element discretization for above set of equations, a linear system of algebraic equations with respect to the unknown nodal values a on the boundary can be expressed in an abbreviated form

$$A(\lambda)\,a = 0 \tag{9}$$

where λ is the non-dimensional frequency parameter associated with the frequency parameter ω of the system. Since the singularity of the fundamental solution W_i^* in equations (8) is strong, an auxiliary boundary technique has been employed to calculate the influence matrices $A(\lambda)$.

In order to determine the natural frequency, the determinant of the coefficient matrix $A(\lambda)$ is zero, which can be solved by the step-by-step search technique(Bezine, 1988).

4. The identification procedure

The identification of the boundary condition may be viewed as an optimization problem. The objective function to be minimized is written as a least-squares difference between the computed frequencies $\lambda_i(z)$ and the measured natural frequencies $\bar{\lambda}_i$ of the system.

$$f(z) = \sum_{i=1}^{I} \phi_i \left[\frac{\lambda_i(z) - \bar{\lambda}_i}{\bar{\lambda}_i}\right]^2 \tag{10}$$

Where ϕ_i is an arbitrary weighting parameter to change the sensitivity in the minimization process; and the design vector z is defined in terms of some undetermined parameters that can completely describe the boundary environment of system. For the minimization of the objective function $f(z)$, a conjugate gradient technique of optimization was adopted. The method starts with an initial guess z_0, and generates the improved value as

$$z_{k+1} = z_k + l_k \boldsymbol{\theta}_k \tag{11}$$

where k is the iteration number, θ_k is the search direction that is modified in each step by the conjugate gradient technique, and l_k is the step-length along the search