an n-dimensional mass vector with m_I being the mass of the ith storey unit; K and $C = (n \times n)$ stiffness and damping matrices, respectively; D = an $(n \times r)$ location matrix; $u(t)$ = an r-vector consisting of r control forces.

In the state space, Equation (1) can be written as

$$\dot{Z}(t) = AZ(t) + Bu(t) + E\ddot{x}_g(t) \tag{2}$$

where $Z(t)$ is a $(2n \times 1)$ state vector; A is a $(2n \times 2n)$ system matrix; B is a $(2n \times r)$ control matrix and E is a $(2n \times 1)$ excitation vector, respectively, given by

$$Z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I_n \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ -M^{-1}m \end{bmatrix} \tag{3}$$

Since the control system described by equation (2) is linear and time-invariant, its analytical solution can be obtained as (Wang, 2004).

$$Z(t) = e^{A(t-t_0)}Z(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau + \int_{t_0}^{t} e^{A(t-\tau)}E\ddot{x}_g(\tau)d\tau \tag{4}$$

Suppose all information for the on-line calculation of control forces is sampled with period T. Between two consecutive sampling points, kT and $(k+1)T$, the best available information about the external excitations is $\ddot{x}_g(kT)$. Therefore, the external excitations are sampled as zero-order hold and are thus assumed to be constant between two consecutive sampling instants. During real-time control, the control forces are calculated once every sampling period. Then the discrete-time control signals are converted into zero-order hold continuous-time signals and applied to the structure. The control forces are constant between two consecutive sampling instant. Let $t_o = KT$ and $t = (k+1)T$, Equation (4) becomes

$$Z[(k+1)T] = e^{AT}Z(kT) + \int_{kT}^{(k+1)T} e^{A[(k+1)T-\tau]}Bu(kT)d\tau + \int_{kT}^{(k+1)T} e^{A[(k+1)T-\tau]}E\ddot{x}_g(kT)d\tau \tag{5}$$

Making variable substitution of $\eta = (k+1)T - \tau$, Equation (5) becomes

$$Z[(k+1)T] = e^{AT}Z(kT) + \int_0^T e^{A\eta}Bu(kT)d\eta + \int_0^T e^{A\eta}E\ddot{x}_g(kT)d\eta \tag{6}$$

As a whole, it is more logical and more realistic for the control system to be modeled in a discrete-time fashion as

$$Z(k+1) = A_d Z(k) + B_d u(k) + E_d \ddot{x}_g(k) \tag{7}$$

where $A_d = e^{AT}$ is a $(2n \times 2n)$ discrete-time system matrix; $B_d = \int_0^T e^{AT}Bd\eta$ is a $(2n \times r)$ discrete-time control matrix and $E_d = \int_0^T e^{AT}Ed\eta$ is a $(2n \times 1)$ discrete-time excitation vector.

3. Design of discrete-time variable structure control involving fuzzy adaptive regulation of reaching law

In the above section, the continuous-time system has been discretized into the standard discrete form. The new DVSC method involving fuzzy adaptive regulation of reaching law for the system will be investigated in this section.

For the DVSC system, the state trajectory starting from any initial state seldom arrives at the switching surface exactly because the state of the system is a discrete-time sequence at interval of sampling period. In state space, the state trajectory shows a point range and not a continuous-time curve, which implies that switching seldom occurs on the switching surface. So the VSC for discrete-time systems is different from the traditional VSC for continuous-time systems. Gao (1996) has made detailed research on the DVSC, and the quasi-sliding mode and the quasi-sliding mode band have been