

rigorously defined in his research. From Gao (1996), the DVSC should have the following three attributes:

(a) Starting from any initial state, the trajectory will move monotonically toward the switching surface and cross it in finite time.

(b) Once the trajectory has crossed the switching surface the first time, it will cross the surface again in every successive sampling period, resulting in a zigzag motion about the switching surface.

(c) The size of each successive zigzagging step is non-increasing and the trajectory stays within a specified band.

The motion of the DVSC system satisfying attributes (a) and (b) is called quasi-sliding mode, whereas the specified band which contains the quasi-sliding mode is called the quasi-sliding mode band which is defined by Reference (Gao, 1996).

$$S^\Delta = \{Z(k) \in R^n \mid -\Delta < \bar{S}(k) < +\Delta\} \tag{8}$$

Where 2Δ is the width of the band. When , the quasi-sliding mode becomes an ideal quasi-sliding mode. In a practical situation, the switching surface can be determined by using the ideal-quasi-sliding mode, and the controller can be obtained by using the discrete-time reaching condition. Designs of the switching surface and the controller are as follows.

3. 1. Design of switching surface

The first step of using the theory of DVSC system design is to determine the switching surface. In the design of the switching surface, the external excitation $\ddot{x}_g(k)$ is neglected; however it is taken into account in the design of the controller (Yang, 1995; Yang, 1995; Yang, 1996;

$$u(k) = -(\Theta B_d)^{-1} \{ \Theta A_d Z(k) + (qTI - I)\Theta Z(k) + \Theta E_d \ddot{x}_g(k) + \varepsilon TI \cdot \text{sgn}[S(k)] \} \tag{11}$$

For the DVSC based on the discrete-time exponential reaching law, the dynamic quality of DVSC system in the reaching mode can be controlled by the choice of the parameters (ε and q) of the difference equation. Generally, the “ q ”parameter influences the reaching velocity at which the control system gets to the switching surface. The control system can arrive at the switching surface rapidly when the “ q ”parameter has acquired a bigger value, especially, when the “ q ”parameter almost is $1/T$.

Yang, 1996; Zhao *et al.*, 2000; Cai and Huang, 2002). Consider a linear switching surface. For simplicity, let $S(k)=[S_1(k),S_2(k),\dots,S_r(k)]^T$ be an r -dimensional switching surface with r sliding variables, $S_1(k),S_2(k),\dots,S_r(k)$, given by

$$S(k) = \Theta Z(k) = 0 \tag{9}$$

where $\Theta = an(r \times 2n)$ matrix to be determined such that the motion on the sliding surface is stable. There are several approaches which can be used to determine the matrix Θ , such as the pole assignment method, the linear quadratic regulator (LQR) method and the null assignment method (Yang, 1995; Yang, 1995; Yang, 1996; Yang, 1996; Zhao *et al.*, 2000; Cai and Huang, 2002; Gao, 1996, Ou, 2003).

3. 2. Design of discrete-time variable structure controller

In the theory of DVSC, the controllers are designed to drive the nonlinear system-state trajectory on a specified and user-chosen surface in the state-space (phase plane). Having forced the states on the switching surface, the controller forces the system state trajectory to remain on the surface for all subsequent time and slides towards the origin. To achieve the above goal, Gao (1996) proposed an exponential reaching law to be used for the controllers design in early nineties and it can be expressed as

$$S(k+1) = (I-qTI)S(k)-\varepsilon TI \text{sgn}[s(k)] \tag{10}$$

where T is the sampling period; $\varepsilon > 0$ and $q < 0$ are both positive real constant, and q is restricted by $qT < 1$.

Upon substituting Equation (7) into Equation (10), the closed-loop (feedback) controller based on the exponential law is given as :

In comparison with the “ q ”parameter, the “ ε ”parameter mainly influences the control system performance to resist the outer disturbance. The control system can resist the outer disturbance more efficiently when the “ ε ”parameter has acquired a bigger value, but it may render the control system unstable because of the undue chattering effect in this condition.

Applications of fuzzy logic (FL) are quite easy although FL has a very broad mathematical foundation.