# ESTIMATION OF MATERIAL PARAMETERS OF FRP COMPOSITE PLATES FROM VIBRATION TEST DATA USING FINITE ELEMENT MODEL UPDATING

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## Introduction

For safe and efficient operation of structures made of fibre reinforced plastics (FRP), its dynamic behaviour must be predicted as realistically as possible. Such behaviour is functions of material and geometric properties, boundary conditions and applied forces. For the determination of material property parameters quasi-static testing of samples is destructive and slow. Due to differing manufacturing and curing processes followed in practice, the actual material property parameters may differ significantly from those specified by the manufacturers. It would be better if the static testing procedure can be supplemented by an efficient methodology.

Increasing ability of the finite element techniques and tremendous advancements in vibration modal testing, promising indirect inverse techniques such as finite element model updating have been proposed [1] and now in a stage of application to real problems. This is basically the iterative minimization of the discrepancies (objective function) of the modal parameters (or any response parameters) obtained from modal testing and finite element analysis to estimate various physical parameters, such as the material properties. One very important problem is to estimate the global average material property parameters of FRPs.

## Experiment

An experimental modal testing was conducted using impact excitation from a hammer to excite a number of grounded square FRP plates and a piezoelectric accelerometer to measure responses. These are Fourier transformed using FFT analyser and the frequency response functions are computed. Natural frequencies, mode shapes and the modal damping parameters are extracted employing modal analysis implementing a developed software [2]. Altogether, five square FRP plates made of Chopped Strand Mat (CSM) and Woven Roving (WR) were tested (Table 1)

## Finite Element Modelling

A computer code has been implemented using an eight noded five degrees-of-freedom per node quadratic isoparametric plate bending element including first order shear deformation for modelling the plate [3]. A simultaneous iteration algorithm has been implemented [4] to calculate the natural frequencies and mode shapes. The results are also checked using a commercially available finite element package, NISA [5] implementing the same isoparametric element.

Table 1. Geometrical properties of the plates (333 mm sq.)

| Plate No. | Lay-up | Number of plies | Layerwise thickness (mm) | Stacking sequence |
|---|---|---|---|---|
| 1 | WR | 5 | 0.54 | 0/0/0/0/0 |
| 2 | CSM | 5 | 1.32 | 0/0/0/0/0 |
| 3 | WR | 5 | 0.61 | 0/45/0/45/0 |
| 4 | WR | 4 | 0.65 | 30/-30/30/-30 |
| 5 | CSM | 3 | 0.81 | 0/0/0 |

## Estimation of Material Constants using Model Updating Techniques

The model updating method comprises two classical approaches viz. Inverse Eigensensitivity Method (IEM) [6] and Levenberg-Marquardt Algorithm (LMA) [7] and one naive approach, viz. Genetic Algorithm (GA). The purpose of the present investigation is to study the ability of a few established model updating methodologies in presence of real noisy measurements to estimate the material constants of FRP plates verified from static test data. There are altogether four independent globally averaged material constants - E1, E2, G12 and PR12, out of which E1 and E2 are considered as same here. A modal sensitivity analysis is carried out with respect to these parameters

## Results and Discussions

Both the convergence and the uniqueness of estimated parameters are studied. For *Plate 1*, modes 1,2,4 and 5 are included for the evolution of fitness values using GA (Fig.1). The use of IEM with only three modes, viz. 1,2 and 4 is found to be inadequate for convergence of all the three parameters. However, the Young's modulus always converges uniquely to a particular value (11.27 GPa) from all the randomly selected starting points (Fig. 2). For LMA, all the three parameters converge uniquely to a particular set of values (Fig.3). Similar curves have been obtained for other plates and have not been presented for brevity. Table 2 and 3 summarises the results.

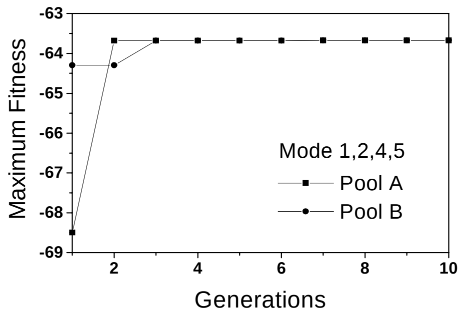


Fig.1 Typical fitness values for PLATE 1 using GA

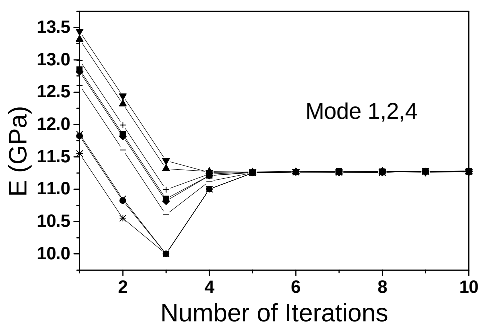


Fig. 2 Typical Convergence of E for PLATE 1 using IEM

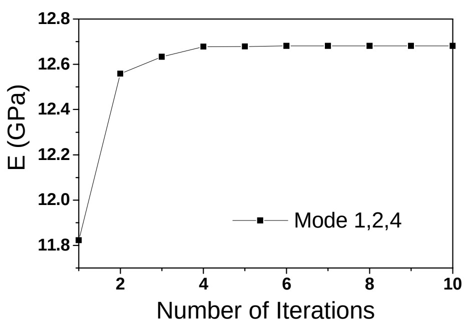


Fig. 3 Typical Convergence of E for PLATE 1 using LMA

Table 2. Estimated values of the elastic parameters using different numerical techniques

| | | E (GPa) | G12(GPa) | PR12 |
|---|---|---|---|---|
| Plate 1 | GA | 12.67 | 2.435 | 0.17 |
| | IEM | 11.27 | - | - |
| | LMA | 12.681 | 2.394 | 0.175 |
| Plate 2 | GA | 7.62 | 2.436 | 0.29 |
| | IEM | 6.961 | 2.958 | 0.32 |
| | LMA | 7.62 | 2.436 | 0.30 |
| Plate 3 | GA | 11.29 | 3.687 | 0.21 |
| | IEM | 11.43 | 3.425 | 0.215 |
| | LMA | 11.18 | 3.364 | 0.20 |
| Plate 4 | GA | 9.83 | 4.16 | 0.195 |
| | IEM | 10.1 | 3.95 | 0.175 |
| | LMA | 9.83 | 4.16 | 0.195 |
| Plate 5 | GA | 6.39 | 4.27 | 0.30 |
| | IEM | 7.77 | 3.255 | 0.279 |
| | LMA | 6.39 | 4.27 | 0.30 |

Table 3. Elastic constants of the plates obtained from static testing

| Plate | E (GPa) | G12 (GPa) | PR12 |
|---|---|---|---|
| 1 | 12.69 | 2.56 | 0.2 |
| 2 | 8.32 | 3.19 | 0.3 |
| 3 | 10.33 | 4.308 | 0.2 |
| 4 | 10.33 | 4.308 | 0.2 |
| 5 | 7.79 | 2.997 | 0.3 |

## Conclusions

Two gradient based methods, namely the inverse eigensensitivity method and Levenberg-Marquardt algorithm and one evolutionary algorithm, namely a real coded genetic algorithm have been tried successfully to estimate all in-plane material parameters of FRP plates in grounded condition. The use of different algorithm, the formulation of which are basically different and their repeatability gives some confidence through the case studies. The convergence and uniqueness of results are encouraging along with some observed limitations. Nevertheless, the results are finally verified by static characterization test. The successful estimation of these parameters are case dependent and varies with the lay-up and stacking sequences thereby necessitates separate case studies, as is done here.

## References


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