MACROSCOPIC CONSTITUTIVE LAW FOR ELASTOPLASTIC COMPOSITE MULTILAYERED MATERIALS

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Introduction

A composite multilayered material is a media with periodic microstructures that is widely used in engineering applications. Due to the variety of available composite multilayered materials, it is very expensive to obtain material characterizations of various multilayered materials using an experimental approach. Moreover, computational methods such as the conventional finite element method are very time-consuming due to intensive discritization of multilayered materials. A potential approach to obtain a macroscopic constitutive relation for these materials is the homogenization method, which ensures the uniformity of micro-stress and strain tensors in each periodic constituent. As a systematic mathematical approach, the homogenization method has several attractive features [1]. It is capable of dealing with microstructures in a periodic order, which may be computationally too intensive to be discretized into a conventional finite element method. Furthermore, the method with certain modifications can be easily incorporated into conventional numerical tools such as the finite element method . In this paper a macroscopic constitutive model for multilayered materials has been developed using the homogenization approach assuming, elasto-plastic behavior for constituents. First, the mathematical formulation of the macroscopic constitutive model is presented. Next, the numerical implementation of this model in a finite element program is described. The last part concerns the validation of proposed model.

Model Descriptions

The basic cell of the multilayered material (Y) is illustrated in Fig. 1. It is composed of two constituents Y^1 and Y^2 whose volume fractions are W^1 and W^2 ($W^1+W^2=1$), respectively. The boundary of the basic cell (Y) is designated as ∂Y. The macro-stress (Σ) and strain (E) tensors define as the average value over the basic cell of the micro-stress (σ) and strain (e) tensors, respectively:

$$\Sigma = \frac{1}{|Y|}\int_Y \sigma \, dY \qquad (1a) \qquad E = \frac{1}{|Y|}\int_Y e(u)\, dY \qquad (1b)$$

where the vector $u(y_1, y_2, y_3)$ is the displacement field at the material point in question.

Assuming the basic cell is subjected to the macro-strain tensor (E), the macro-stress tensor (Σ) related to E can be obtained by the resolution of the elasto-plastic problem described by the following relations:

(i) Equilibrium equations: $\operatorname{div} \sigma^i = 0$

(ii) Constitutive equations: The behavior of each constituent is assumed to be governed by the elasto-plastic constitutive relation with a non-associated flow rule. The constitutive relation of the i^{th} constituent is defined by the elasticity tensor a^i, the yield function f^i, and the plasticity potential g^i. The micro-stress tensor (σ^i) in the i^{th} constituent can be expressed as follows:

$$\sigma^i = a^i : \left(e^i - \left(e^p\right)^i\right) \qquad (2)$$

where e^i and $(e^p)^i$ denote the total and plastic micro-strain tensors in the i^{th} constituent.

(iii) Imposed macro-strain: $E = \frac{1}{|Y|}\int_Y e(u)\, dY$

(iv) Periodicity conditions: e(u) is Y-periodic and $\sigma.n$ is Y-antiperiodic

The resolution of this elasto-plastic problem enables the micro-stress to be calculated to determine the macro-stress tensor (Σ).

Application of the homogenization approach to multilayered materials [2, 3] permits determination of the micro-strain tensors (e^1 and e^2) in terms of the macro-strain tensor (E), the plastic microstrain tensors ($(e^p)^i$, i = 1,2) and the plastic component of the relative displacement at the interface (Δu_t^p) through the following:

$$e^i = C_E^i : E + \sum_{j=1}^{2} C_j^i : \left(e^p\right)^j + C_{int}^i : \Delta u_t^p \otimes n_{int} \quad (3)$$

where the symbol $\otimes$ defines: $(u \otimes v)_{ij} = \frac{1}{2}(u_i v_j + u_j v_i)$,

C_E^i represents the strain concentration tensor in the elastic domain. C_j^i and C_{int}^i are fourth order plastic strain concentration tensors. The micro-stress tensors can be computed as:

$$\sigma^i = S_E^i : E - \sum_{j=1}^{2} S_j^i : (e^p)^j + S_{int}^i : \Delta u_t^p \otimes n \qquad (4)$$

Expressions of tensors S_E^i, S_j^i and S_{int}^i are given as follows:

$$S_E^i = a^i : C_E^i \qquad S_j^i = a^i : (I\delta_j^i - C_j^i) \qquad S_{int}^i = a^i : C_{int}^i$$

where I and δ_j^i denote the fourth order unit tensor and the Kronecker symbol, respectively.

The expression of the macro-stress tensor can be obtained from equation (1a) and (4) through:

$$\Sigma = a^{hom} : E - \sum_{i=1}^{2} \sum_{j=1}^{2} W^i S^i_j : (e^p)^j + S^i_{int} : \Delta u^p_t \otimes n_{int} \quad (5)$$

where a^{hom} denotes the homogenized elasticity tensor,

$$a^{hom} = \sum_{i=1}^{2} W^i S^i_E$$

Equation 5 shows that the macroscopic constitutive relation involves the micro-plastic strains ($(e^p)^i$, i = 1,2) and the plastic component of the relative displacement at the interface (Δu^p_t) as hardening parameters.

Numerical Implementation

The macroscopic constitutive model presented in the previous section was implemented in a finite element program using the classical Newton-Raphson scheme for the resolution of elasto-plastic problems. The stiffness matrix was calculated using the homogenized elasticity tensor a^{hom}. For a given loading step, the increment of the macro-strain tensor (ΔE) was derived from the field displacement. Implementation of the macroscopic constitutive model required the elaboration of certain subroutines, which permitted the determination of the model response to the increment of the macro-strain tensor (ΔE). The micro-stress increment was computed assuming an elastic behavior for the multilayered material ($\Delta\sigma^i = S^i_E : \Delta E$, $\tau_{int} = K_{int}\Delta u_t$). Yield criteria were then calculated for the two constituents and the interface. If any yield criterion was violated, plastic strains were computed using the consistency condition and the plastic flow rules. The out of equilibrium forces were calculated and distributed following the classical method used in elasto-plastic modeling.

Application example: Out-of-Plane Tension Paths

The study of the influence of the interface on the response of a basic cell presenting a composite multilayer material to tension paths imposed in α=30º directions with respect to the y_1 axis (Fig. 2a) using the proposed model is described in this section. The response of the composite material is illustrated in Figs. 2b, which show the evolution of the deviator of the macroscopic stress tensor in terms of the macroscopic strain in the direction of loading (E_α). It can be observed that the presence of an imperfect interface largely affected the behavior of the composite material. Failure occurred by plastification of the matrix material and interface slipping, which appeared at the same time. The presence of the imperfect interface reduced the resistance of the composite material by about 58%.

Conclusion

A new macroscopic constitutive law for multilayered materials with imperfect interfaces using the homogenization method was described in this study. By integrating the advantages of the conventional finite element formulation and the homogenization method, the proposed technique can be used to model composite multilayered structures. The proposed model can be easily implemented in finite element programs and used for calculation of composite multilayered structures with imperfect condition. This model was shown to be capable of dealing with microstructures in a periodic order, which may be too computationally intensive to be discretized using a conventional finite element model.

References

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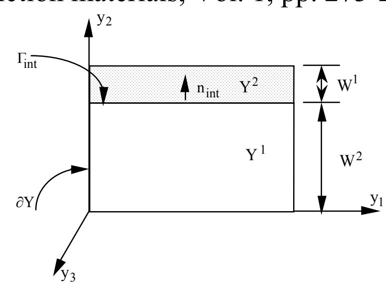
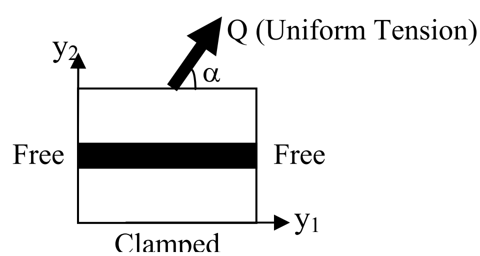


Fig. 1: Basic cell of a multilayered material with two constituents.



(a)

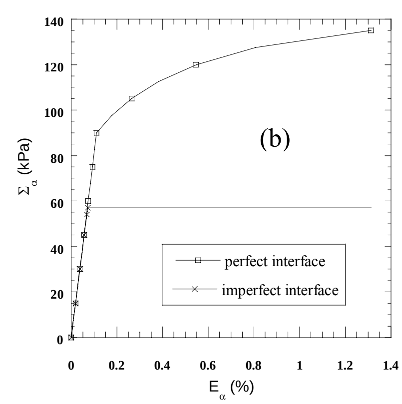


Fig. 2a-b: Response of the multilayered material to out-of-plane tension loading (a) Schematic of the configuration (b) α =30º.