MACROSCALE HEAT TRANSFER IN NANOFLUIDS

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Introduction

Choi coined the term "nanofluids" for the fluids with nanoelements (nanoparticles, nanotubes or nanofibers) suspended in them. Recent experiments on nanofluids have shown substantial increases in thermal conductivity and convective heat transfer coefficient with low particle volume concentrations compared with liquids without nanoparticles or larger particles, strong temperature dependence of thermal conductivity, and substantial increases in critical heat flux (CHF) in boiling heat transfer. These characteristics make them very attractive for a large number of industries such as transportation, electronics, defense, space, nuclear systems cooling and biomedicine.

The study of nanofluids is still in its infancy. The precise nature and mechanism of the significant improvement of thermal performance are still not known. There is also a lack of agreement between experimental results and between theoretical models. The fact that the enhancement in thermal properties comes from the presence of nanoparticles has directed research efforts nearly exclusively towards thermal transport at nanoscale. The classical energy equation has been postulated as the macroscale model but without adequate justification. Thermal conductivity and convective heat transfer coefficient are a macroscale phenomenological characterization of heat transfer and the measurements are not performed at the nanoscale, but rather at the macroscale. Therefore, interest should focus not only on what happens at the nanoscale but also on how the presence of nanoelements affects the heat transport at macroscale.

We attempt to rigorously develop a macroscale heat transfer model in nanofluids by scaling-up the microscale model for the heat transfer in the nanoparticles and in the base fluids. The approach for scaling-up is the volume averaging with help of multiscale theorems.

Heat Transfer in Nanofluids

The microscale model for heat transfer in nanofluids is well-known. It consists of the field equation and the constitutive equation. The field equation comes from the conservation laws of mass, momentum and energy. The commonly-used constitutive equation includes the Fick law of diffusion, the Newton law of viscosity and the Fourier law of heat conduction.

For transport in nanofluids, the macroscale is a phenomenological scale that is much larger than the microscale and much smaller than the system length scale. Interest in the macroscale rather than the microscale comes from the fact that a prediction at the microscale is complicated due to the complex microscale structure of nanofluids, and also because we are usually more interested in large scales of transport for practical applications. Existence of such a macroscale description equivalent to the microscale behavior requires a good separation of length scales and has been well discussed in [2, 3].

To develop a macroscale model of heat transfer in nanofluids, the method of volume averaging starts with a microscale description. Both conservation and constitutive equations are introduced at the microscale. The resulting microscale field equations are then averaged over a representative elementary volume (REV), the smallest differential volume resulting in statistically meaningful local averaging properties, to obtain the macroscale field equations. In the process of averaging, the *multiscale theorems* are used to convert integrals of gradient, divergence, curl, and partial time derivatives of a function into some combination of gradient, divergence, curl, and partial time derivatives of integrals of the function and integrals over the boundary of the REV [3].

Consider heat transfer in nanofluids with the base fluid and the nanoparticle denoted by f-and p-phases, respectively. As a two-component mixture (base fluid + nanoparticles), its microscale model is available in [4]. Applying the volume averaging and multiscale theorems to scale-up the microscale model yields a macroscale model, where the energy equation reads for isotropic nanofluids with constant physical properties of the base fluid and nanoparticles,

$$\frac{\partial \langle T_i \rangle^i}{\partial t} + \tau_q \frac{\partial^2 \langle T_i \rangle^i}{\partial t^2} + \langle \mathbf{v}_i \rangle^i \cdot \nabla \langle T_i \rangle^i = \alpha \, \Delta \langle T_i \rangle^i +$$
$$\alpha \tau_T \frac{\partial}{\partial t} \left(\Delta \langle T_i \rangle^i \right) + \frac{\alpha}{k} \left[F(\mathbf{r},t) + \tau_q \frac{\partial F(\mathbf{r},t)}{\partial t} \right] \qquad (1)$$

where

$$\tau_q = \frac{\gamma_f \gamma_p}{ha_v(\gamma_f + \gamma_p)}, \qquad (2)$$

$$\tau_T = \frac{\gamma_f k_{pp} + \gamma_p k_{ff}}{ha_v(k_{ff} + k_{pp} + k_{fp} + k_{pf})}, \qquad (3)$$

$$k = k_{ff} + k_{pp} + k_{fp} + k_{pf}, \qquad (4)$$

$$\alpha = \frac{k_{ff} + k_{pp} + k_{fp} + k_{pf}}{\gamma_f + \gamma_p}, \qquad (5)$$

$$F(\mathbf{r},\ t) + \tau_q \frac{\partial F(\mathbf{r},\ t)}{\partial t} = \frac{1}{a_v h}\Big\{ \left(k_{fp} k_{pf} - k_{ff} k_{pp}\right) \Delta^2 \langle T_i \rangle^i$$
$$+ \left[\gamma_f \frac{\partial}{\partial t}\left(\mathbf{u}_{pp} \cdot \nabla \langle T_i \rangle^i \right) + \gamma_p \frac{\partial}{\partial t}\left(\mathbf{u}_{ff} \cdot \nabla \langle T_i \rangle^i \right) \right]$$
$$- \Big\{ k_{pp} \Delta \left(\mathbf{u}_{ff} \cdot \nabla \right) k_{ff} \Delta \left(\mathbf{u}_{pp} \cdot \nabla \right) - k_{pf} \Delta \left(\mathbf{u}_{fp} \cdot \nabla \right)$$
$$- k_{fp} \Delta \left(\mathbf{u}_{pf} \cdot \nabla \right) \Big\} \langle T_i \rangle^i$$
$$- \left[\left(\mathbf{u}_{ff} \cdot \nabla \right)\left(\mathbf{u}_{pp} \cdot \nabla \right) - \left(\mathbf{u}_{fp} \cdot \nabla \right)\left(\mathbf{u}_{pf} \cdot \nabla \right) \right] \langle T_i \rangle^i \Big\}$$
$$\qquad (6)$$

Here the index i can take f or p; $\langle T_i \rangle^i$ is the i-component intrinsic average temperature; γ_f and γ_p are the f-component and p-component effective thermal capacities, respectively; k_{ff} and k_{pp} are the thermal dispersion coefficients of the f- and p-components, respectively; k_{fp} and k_{pf} are the cross thermal dispersion coefficients of the two components; h and a_v are the film heat transfer coefficient and the interfacial area per unit volume, respectively; $\langle \mathbf{v}_i \rangle^i$ is the i-component intrinsic average velocity; and the four The velocity-like coefficients satisfy

$$\mathbf{u}_{ff} + \mathbf{u}_{fp} + \mathbf{u}_{pf} + \mathbf{u}_{pp} = 0. \qquad (7)$$

The models for ha_v, four thermal dispersion coefficients and four velocity-like coefficients have

also been obtained in the present work. Note also that k_{ff}, k_{pp}, k_{fp} and k_{pf} contain contributions from both conductive and convective transports.

Results and Discussion

Equation (1) is a dual-phase-lagging energy equation with τ_q and τ_T as the phase lags of the heat flux and the temperature gradient, respectively [5, 6]. In Eq. (1) $F(\mathbf{r},t)$ is the volumetric heat source. k, ρc and α are the effective thermal conductivity, capacity and diffusivity of nanofluids, respectively. Therefore, the presence of nanoparticles shifts the Fourier-type heat transfer in the base fluid into the dual-phase-lagging heat transfer in nanofluids at the macroscale. This is significant because all results regarding dual-phase-lagging heat transfer can thus be applied to study heat transfer in nanofluids.

It is interesting to note that the non-traditional velocity-like terms appear only in the source term in Eq. (1). Therefore, the microscale physics does not manifest itself as the macroscale convection.

The presence of nanoparticles gives rise to variations of thermal capacity, conductivity and diffusivity, which are given by, in terms of ratios over those of the base fluid,

$$\frac{\rho c}{(\rho c)_f} = (1 - \varphi) + \varphi \frac{(\rho c)_p}{(\rho c)_f}, \qquad (8)$$

$$\frac{k}{k_f} = \frac{k_{ff} + k_{pp} + k_{fp} + k_{pf}}{k_f}, \qquad (9)$$

$$\frac{\alpha}{\alpha_f} = \frac{k}{k_f} \frac{(\rho c)_f}{\rho c}. \qquad (10)$$

Therefore, $\rho c / (\rho c)_f$ depends *only* on the volume fraction of nanoparticles (φ) and the nanoparticle-fluid capacity ratio. However, both k/k_f and α/α_f are affected by the geometry, property and dynamic process of nanoparticle-fluid interfaces, and convective process. This dependency causes the most difficulty because it is the least precisely known feature of a nanofluid. Consider

$$\frac{\tau_T}{\tau_q} = 1 + \frac{\gamma_f^2 k_{pp} + \gamma_p^2 k_{ff} - \gamma_f \gamma_p (k_{fp} + k_{pf})}{\gamma_f \gamma_p k}. \qquad (11)$$

It can be larger, equal or smaller than 1 depending on the sign of $\gamma_f^2 k_{pp} + \gamma_p^2 k_{ff} - \gamma_f \gamma_p (k_{fp} + k_{pf})$. Therefore, by the condition for the existence of thermal waves that requires $\tau_T / \tau_q < 1$ [6, 7], we may have thermal waves in nanofluid heat transfer when

$$\gamma_f^2 k_{pp} + \gamma_p^2 k_{ff} - \gamma_f \gamma_p (k_{fp} + k_{pf}) < 0. \qquad (12)$$

Note also that for heat transfer in nanofluids there is a time-dependent source term $F(\boldsymbol{r}, t)$ in the dual-phase-lagging heat conduction [Eqs. (1) and (6)]. Therefore, the resonance can also occur.

Therefore, the molecular physics and the microscale physics (interactions between nanoparticles and base fluids at the microscale in particular) manifest themselves as heat diffusion and thermal waves at the macroscale, respectively. Their overall macroscopic manifestation shifts the Fourier-type heat transfer in the base fluid into the dual-phase-lagging heat transfer in nanofluids. Depending on factors like material properties of nanoparticles and base fluids, nanoparticles' geometrical structure and their distribution in the base fluids, and interfacial properties and dynamic processes on particle-fluid interfaces, the heat diffusion and thermal waves may either enhance or counteract each other. Consequently, the heat transfer may be enhanced or weakened by the presence of nanoparticles.

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