A MODIFIED ENERGY-BALANCE MODEL TO PREDICT LOW-VELOCITY IMPACT RESPONSE FOR SANDWICH COMPOSITES

C. C. Foo[a], L. K. Seah[b], and G. B. Chai[b]
[a]Institute Of High Performance Computing, 1 Fusionopolis Way, #16-16 Connexis, Singapore 138632
[b]School of Mechanical & Aerospace Engineering, Nanyang Technological University, Singapore

INTRODUCTION

Composite sandwich structures are widely used in lightweight construction because of their high specific strengths and stiffnesses. However such structures are susceptible to low-velocity impact damage, which reduces the structural stiffness and strength [1].

The spring–mass models and energy-balance models are two of the few analytical solutions proposed to study the impact response of composite sandwich structures [1]. But these elastic models cease to be valid after damage initiation and are unable to model damage propagation [1]. Moreover, the energy-balance model only yields the maximum impact force but not the load history. Modified spring-mass models have been proposed to account for damage but they depend strongly on unknowns that have to be determined experimentally [2].

Here, the energy-balance model is coupled with the law of conservation of impulse-momentum to extend its validity beyond the elastic regime. Closed-form solutions were first derived for three parameters that described the plate's structural behaviour, namely, the plate's elastic structural stiffness, the critical load at the onset of damage, and the reduced stiffness after damage. These three parameters were then included in the modified model to derive load and deflection histories for the sandwich plate. Under low-velocity impact, the plate essentially deforms under quasi-static loading.

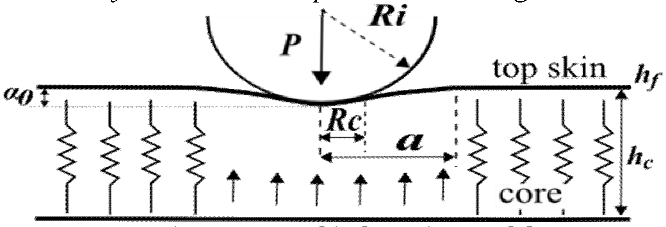


Figure 1: Local indentation model.

ANALYTICAL MODEL

The elastic response of the plate is first considered. The local and global responses are decoupled and any interaction between the two is ignored, so that these stiffnesses can be determined separately. The principle of minimum total potential energy is used here to derive the elastic local stiffness, K_{loc}, of a clamped circular sandwich panel indented by a spherical indentor at its middle (Fig.1). The top skin is modelled as a plate resting on the core subjected to a concentrated load, with membrane stretching of the facesheet neglected for small indentation. The skin is elastic, while the core is modelled as an elastic Winkler foundation. By considering the bending energy of the orthotropic skin, the strain energy due to core deformation, and the work done by the point load, it can be shown that the elastic stiffness of the plate is,

$$K_{loc} = 12.98\sqrt{D_f E_{33c} / h_c} \qquad \text{-- (1)}$$

where D_f is the bending stiffness of the top skin, E_{33c} and h_c is the core modulus and core height, respectively.

The bending and shear stiffnesses of the plate, which constitute the global stiffness, are derived [3]. The shear stiffness of the plate, K_s, is derived by dividing the central uniformly distributed pressure load, which acts over a contact area with radius R_c, by the shear deflection at the centre of the plate to give [3],

$$K_s = \frac{4\pi G_c (h_f + h_c)^2}{h_c\left(1 + 2\ln(R_p/R_c)\right)} \qquad \text{-- (2)}$$

where G_c is the core shear modulus; h_f is the thickness of the skin; R_p and R_c are the outer and contact radii of the plate, respectively. Using the classical plate theory, the effective bending stiffness of the sandwich plate, K_b, is

$$K_b = \frac{16\pi D'_{sw}}{R_p^2} \qquad \text{-- (3)}$$

where D'_{sw} is the equivalent bending stiffness of the clamped sandwich plate.

At the onset of damage, there is a substantial drop in the stiffness, and several studies have identified core failure to occur at this critical state [4]. Thus the load at failure initiation (P_I) is derived by considering the elastic energy absorbed by the plate up to the onset of core damage. First, the strain energy absorbed by the core, U_I, at initial failure under local indentation is estimated by.

$$U_{core} = \pi\beta^2 R_{ind}^2 U^* \qquad \text{-- (4)}$$

where β is a constant and R_{ind} is the indentor radius. U^* is the energy absorbed by the core per unit area at initial failure, which is derived from flatwise compression tests on bare honeycombs. Next, the elastic strain energy of the top skin at the onset of damage, U_{tfs}, is calculated. This energy comprises the bending and membrane stretching energies (U_b and U_m),

$$U_b = \frac{32 D_f \alpha_0^2}{3a^2}; U_m = 2.59\pi D_f\left(\frac{\alpha_0^4}{a^2 h_f^2}\right) \qquad \text{-- (5)}$$

where α_0 is the local indentation and a is the radius of region of local indentation on the top skin. From the local indentation model (Fig.1), a can be derived: $a=3.214(D_f E_{33c}/h_c)^{0.25}$.

The energy due to local indentation can be expressed as

$$U_c = \int_0^{\alpha_0} P d\alpha_0 = \frac{1}{2} K_{loc}\alpha_0^2 = \frac{P^2}{2K_{loc}} \qquad \text{--(6)}$$

This energy is then equated to the energies sustained by the core and top skin, i.e., $U_c = U_{core} + (U_b + U_m)_{tfs}$. This equation can be solved to find the indentation at the critical state using Eqs. (4), (5), and (6). The load at the critical state, P_1, can be calculated (Eq.6).

It is well-accepted that most of the energy absorbed by the plate during impact is dissipated in the form of damage, and the extent of damage is reflected in the reduction of the plate's stiffness. Thus we assume that the relative loss in energy is related to the decrease in stiffness (K_{dam}/K_0)by the relation

$$\frac{K_{dam}}{K_0} = 1 - \frac{E_{abs}}{E_{imp}} \quad \text{-- (7)}$$

where E_{abs} is the energy absorbed by the specimen, and E_{imp} is the impact energy. This assumption was verified by test data for 11 sandwich configurations of various cores and skins impacted at various energies using Instron Dynatup 8250 impact testing machine (Fig.2). The plates were made of Fibredux 913C-HTA carbon/epoxy skins bonded to HexWeb A1 Nomex honeycombs [4]. The configuration is given as "Laminate orientation/Core cell size/Core height". For e.g., C1/3/15 refers to a plate with laminate orientation C1 and a core of cell size 3mm and 15mm thick.

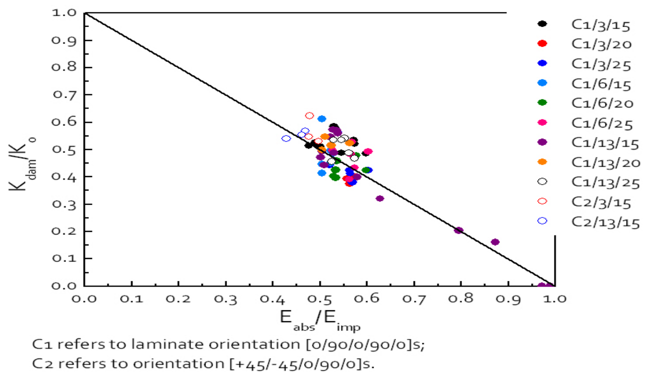


Figure 2: Relative reduction in plate's stiffness as a function of relative loss in impact energy.

Finally, the analytical impact model is used to predict the impact response for sandwich composites. This model comprises the laws of conservation of energy and impulse-momentum,

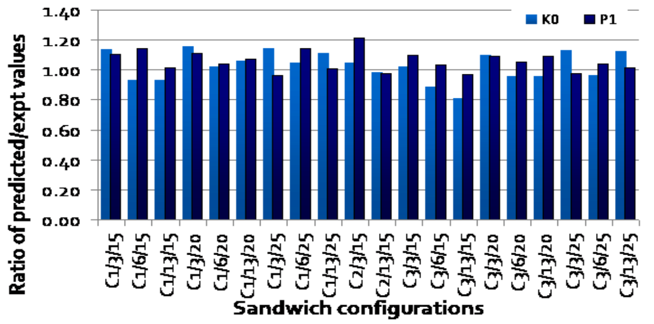
$$U_c + U_{bs} = \frac{p^2}{2K_0} = \frac{1}{2} M_{imp} \left(V_{imp}{}^2 - V(t)^2 \right) \quad \text{-- (8)}$$

$$M_{imp} \left(V_{imp} - V(t) \right) = \int_0^t P dt \quad \text{-- (9)}$$

Where M_{imp} denotes the impactor mass, V_{imp} is the impact velocity and $V(t)$ refers to the velocity at time t. The load and velocity histories are then solved using Eqs. (8) and (9). The deflection of the impactor is then obtained by integration. Once the load reaches P_1, the elastic stiffness K_0 is degraded to K_{dam} to account for damage. The integral on the right-hand side of Eq. (9) is approximated by area under the load history using the trapezoidal rule. No unloading is considered.

RESULTS

Figure 3 shows the ratios of the predicted elastic stiffness (K_0) and critical load at damage initiation (P_1) compared against test results for 20 configurations of sandwich plates loaded under quasi-static indentation (from [4]). The predicted stiffnesses and loads are within 19% and 22% of the test data, respectively.



*C3 refers to skin orientation [+45/0/0/90/0/0/-45]s

Figure 3: Comparison of predicted and experimental values for elastic stiffness (K_0) and damage initiation loads (P_1) for composite plates loaded by indentation [4].

Figure 4 shows the experimental and predicted load-time and load-deflection histories for a composite sandwich plate impacted at 1.8J (from [4]). The load increased up to P_1 and suddenly dropped. Then the load increased to a maximum at a reduced stiffness (K_{dam}). The predicted results agree well, in terms of the critical and peak loads, as well as overall behaviour.

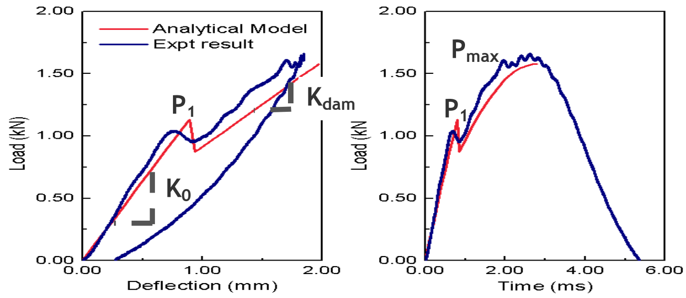


Figure 4: Load-deflection and load-time histories for plate C2/13/15 under 1.8J impact.

CONCLUSION

The modified energy-balance model has been shown to be capable of predicting the low-velocity impact response of a composite sandwich plate by using just three parameters (K_0, P_1, and K_{dam}) to account for elastic response, damage initiation and propagation. This model extends the original energy-balance model which is largely limited to elastic impacts and does not produce load histories.

REFERENCES

[1] S. Abrate. *Impact on composite structures.* Cambridge University Press, 1998.

[2] P. Feraboli. Modified SDOF model for improved representation of impact response of composite plates. *Journal of Composite Materials*, 40:2235-2255, 2006.

[3] D. W. Zhou and W. J. Stronge. Low velocity impact denting of HSSA lightweight sandwich panel. *International Journal of Mechanical Sciences*,48:1031-1045, 2006.

[4] C. C. Foo. Energy absorption characteristics of sandwich structures subjected to low-velocity impact. PhD Thesis. Nanyang Technological University, Singapore, 2009.