COMMON APPROACH TO THE PHASE TRANSITION THEORY BY THE EXAMPLES OF FRACTURE MECHANICS AND THE THEORY OF DETONATION

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Introduction

The evaluation of technology leaded to the new results and problems in mechanics of fracture. In the experiments on dynamic fracture of solids it was obtained that classical approach, based on maximization of Stress Intensity Factor (SIF), has not described these ones. It turned out, that fracture takes place at the subsequent moment after SIF maximization. And the magnitude of rupture impulse can limit some non-zero value, that's contrary to classical theory, but is self-evident. Contrariwise, in the detonation initiation researches it was found that the critical energy, leading to detonation, increases rapidly with the increasing time of energy deposition. In this work the common approach to these problems is proposed. It based on the easy measurable parameters of the given process.

Incubation Time Criterion

In [1] the new Incubation time criterion about the crack propagation was suggested. It can be written follow:

$$\int_{t-\tau}^{\tau} K_1(x,t')dt' \le \tau \cdot K_{1c}, \qquad (1)$$

where K_1 is the Stress Intensity Factor (SIF), K_{1c} is the critical value of K_1 in static case (when a duration of fracture pulse is long enough), τ is the incubation time (a characteristic time of micro-relaxation processes that can be measured experimentally). In [1] it was shown the good correspondence between (1) and the experiments on dynamic and static fracture. But there is no good model about the crack propagation in solids.

The Model of Crack Propagation

For a description of crack propagation let us introduce the damage function $\theta(\bar{r},t) \in [0,1]$ to characterize the relative volume of microdamages in the specimen mass unit in the vicinity of every point $\bar{r} \in \Omega$, where Ω is an arbitrary domain of the matter. Then $\theta = 0$, it correspondences to fully undamaged material, whereas the case $\theta = 1$ correspondences to the local damage at considered scale level.

Follow [2] we will consider the crack as one-dimensional one, and we can write the describing time-variation equation for θ:

$$\frac{\partial \theta}{\partial t} = g(\theta,x,t) + f(\theta,x,t), \qquad (2)$$

where $g(\theta, x, t)$ describes the processes of redistribution of microdamages in the vicinity of x, and $f(\theta, x, t)$ is the source function that describes the macroscopically uniform process of microdamage accumulation.

Let us, for simplicity, suppose the fact, that microdamages can move, and the probability of defect moving from point x within dx at time t is $P(x,t)dx$:

$$g(\theta,x,t) = \psi\left(\int_{-\infty}^{+\infty} \theta(\zeta,t)P(\zeta - x,t)d\zeta - \theta(x,t)\right), \qquad (3)$$

where ψ characterizes the intensity of microdamage redistribution. We assume all directions are equiprobable. So, expanding $\theta(\zeta, t)$ in (3) into a Taylor series by coordinate up to the second order terms, we can rewrite (2):

$$\frac{\partial \theta}{\partial t} = D\frac{\partial^2 \theta}{\partial x^2} + f(\theta,x,t), \qquad (4)$$

where $D = \frac{\psi R^2(t)}{2}$, $R = \sqrt{\int_{-\infty}^{+\infty} \zeta^2 P(\zeta,t)d\zeta}$.

So, non-linear diffusion equation is offered for the crack propagation description. But the question about function $f(\theta, x, t)$ is leaving open.

The Mass Conservation Law and the Microdamage Accumulation

Let us choose a part of considered solid. Its mass before deformation is denoted as m, and volume – V_0. After deformation the volume of accumulated microdamages is V_*. The total volume of damaged part is $V = V_0 + V_*$. Let write the mass conservation law:

$$\frac{d\rho}{dt} = -\rho \cdot div\bar{v}, \qquad (5)$$

where $\rho = \frac{dm}{dV} = \frac{dm}{dV_0}\frac{dV_0}{dV} = \frac{dm}{dV_0}\left(1 - \frac{dV_*}{dV}\right)$ is the local density of solids, $\bar{v}$ is the velocity of material particles. Denoting $\theta = \frac{dV_*}{dV}$ (it may be regarded as the damage function introduced above) and $\rho_0 = \frac{dm}{dV_0}$ (initial density) eq. (5) may be rewritten:

$$\frac{d\theta}{dt} = (1-\theta)\, div\,\bar{v}\,. \qquad (6)$$

If we expand $div\,\bar{v}$ into a power series of θ: $div\,\bar{v} = C_0 + C_1\theta + o(\theta)$, and suppose $C_0 = 0$ (because of the lack of volume extension for the undamaged material) we receive:

$$\frac{d\theta}{dt} = C_1\theta(1-\theta). \qquad (7)$$

Follow [3] we may conclude

$$C_1(x,t) = \frac{1}{K_{1c}\tau}\left(K_1(t) - K_1(t-\tau)\right), \qquad (8)$$

where K_1 is the SIF, K_{1c} is the critical value of K_1, τ is the incubation time.

We may merge the results: the non-linear member of (4) $f(\theta, x, t)$ may be identified with (7). So we receive non-linear diffusion equation for the description of crack propagation:

$$\frac{\partial\theta}{\partial t} = D\frac{\partial^2\theta}{\partial x^2} + \frac{\theta(1-\theta)}{K_{1c}\tau}\left(K_1(t) - K_1(t-\tau)\right). \qquad (9)$$

In [2] it was shown the good correspondence between numerical simulation of crack propagation on basis of (9) and the experiments.

The Energy Criterion of Detonation

In [4] a number of experiments about detonation initiation in the stoichiometric hydrogen–oxygen mixture was presented. The detonation was achieved by electric discharges of different frequencies and amplitudes. It turned out there was the critical value of energy E_{cr} corresponding to the discharge duration (frequency) that the discharge with energy less then E_{cr} couldn't initiate the detonation in the mixture. And there was the time point t_{mix} such that for electric discharges of the duration $t_f < t_{mix}$, the critical energy E_{cr} was independent of the discharge duration and had the constant value E^0. Thus, E^0 is the minimum energy for the detonation of the mixture under given conditions. The experimental results are presented on Fig. 1 (dots).

In our recent work [5] we suggested applying the incubation-time approach to the detonation initiation problem. So the critical condition for the detonation can be written:

$$\int_{t-\tau}^{\tau} U(t')dt' \le \tau \cdot U_c\,, \qquad (10)$$

where $U(t)$ is the power delivered to the mixture, τ is the incubation time inherent in the development of the direct detonation process. The critical energy U_c corresponds to the minimum rate of power supply, which is capable of the detonation initiation. When the rate is smaller then U_c, the formed unsteady over-compressed detonation wave fails to attain the steady-propagation regime. The incubation time τ is connected with the processes preceding the attainment

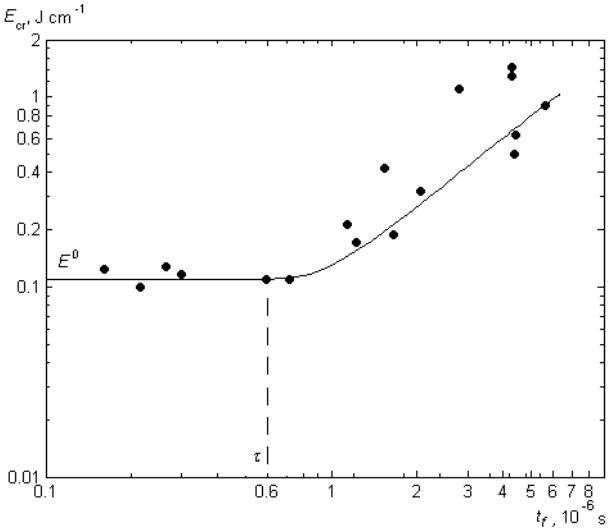

Fig. 1 Critical energy E_{cr} vs the discharge duration t_f: the incubation-time criterion (solid line) and experimental data of [5] (dots)

of the steady regime for the detonation wave propagation, including chemical processes. The time τ is defined as the time interval between the start of the power-supply process (t = 0) and the time of the attainment by the medium of the state that will be obligatorily finished by the formation of the steady detonation wave when the rate of power supply is U_c.

The comparison between incubation-time approach and experiments is presented on Fig. 1. One can notice the good correlation.

Conclusion

As in the case of solids fracture in the case of detonation we may separate two phase: detonated and non-detonated. The common approach to the crack propagation and the detonation initiation problems based on the incubation time criterion and the specimen separation in two phases is presented. The good correlation between experimental data and theory is shown. Further work will trend to the applying non-linear diffusion equation for the description of detonation wave propagation.

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