EXPERIMENTAL AND NUMERICAL COMPUTATIONS OF COMPOSITE NOTCH STRESS INTENSITY FACTORS

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Introduction

Computation of notch stress intensity factors (SIFs) for composite materials is important, since cracks are often initiated at this location. Labossiere and Dunn presented a procedure to calculate mode I and II notch SIFs in anisotropic media using the path-independent H-integral [1]. Qian used a contour integral in conjunction with the finite element method to evaluate the SIFs at the bi-material wedge [2]. Banks-Sills and Ishbir used a conservative integral based on the Betti reciprocal principle to obtain SIFs for a bimaterial notch [3]. Kumagai and Shindo described an experimental and analytical study on the cryogenic fracture behavior of CFRP-woven laminates under tension with a sharp notch [4]. Ju et al. used image correlation experiments to find composite notch SIFs [5].

This study used a least-squares method to find the notch SIFs of anisotropic materials, and the method can be applied to finite element and experimental methods.

Notch SIFs of anisotropic materials

The notch coordinate system is defined in Fig.1, where the origin is located at the notch tip and the center of the notch surface is in the negative x axis, the in-plane y axis is perpendicular to the x axis, r and θ are the in-plane polar coordinates of the point based on the x axis, 1 and 2 axes are the directions of in-plane material symmetry, $+\alpha$ and $-\alpha$ are the angles from the x-axis to notch surfaces anti-clockwisely and clockwisely, respectively, and notch angle γ is $2(\pi-\alpha)$. For composite materials, we define the following four SIFs:

$$\sum_{j=1}^{2} k_I(\delta_j) = \sum_{j=1}^{2}\left(\sqrt{2\pi}r^{-\delta_j}\sigma_{rr}(r=\theta=0)\right) \quad (1)$$

$$\sum_{j=1}^{2} k_{II}(\delta_j) = \sum_{j=1}^{2}\left(\sqrt{2\pi}r^{-\delta_j}\tau_{r_\theta}(r=\theta=0)\right) \quad (2)$$

where δ_j is the j^{th} eigenvalue dependent on material properties and notch angle. The physical meaning of $k_i(\delta_j)$ denotes the ith-mode SIF produced from the singularity of the jth eigenvalue.

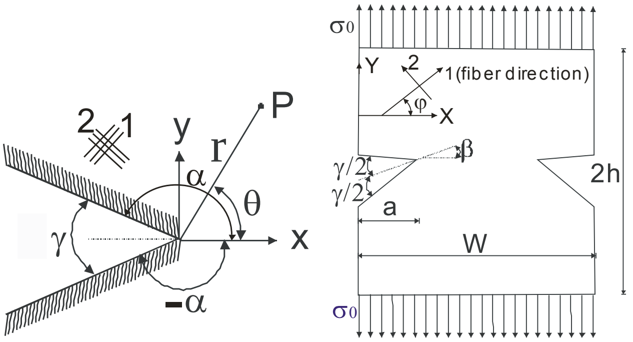

Fig. 1 V-notch and coordinate systems

Numerical and experimental results

The H-integral [1] obtained from the finite element analysis is used to validate the SIFs calculated from the least-squares method. The dimensionless SIFs are defined as:

$$F_I(\delta_j) = \frac{k_I(\delta_j)}{\sigma_0\sqrt{\pi}a^{-\delta_j}}, \quad F_{II}(\delta_j) = \frac{k_{II}(\delta_j)}{\sigma_0\sqrt{\pi}a^{-\delta_j}} \text{ for j=1,2} \quad (3)$$

where σ_0 is the far-field applied normal stress and *a* is the notch length. The error of the least-squares method is defined as:

$$\text{Error} = \frac{\sum_{i=I,II}\left|F_i^H(\delta_1)-F_i^{LS}(\delta_1)\right|+\left|F_i^H(\delta_2)-F_i^{LS}(\delta_2)\right|}{\sum_{i=I,II}\left|F_i^H(\delta_1)\right|+\left|F_i^H(\delta_2)\right|} \quad (4)$$

where $F_i^H(\delta_j)$ are the SIFs obtained from the H-integral and $F_i^{LS}(\delta_j)$ are the those calculated from the least-squares method.

A notched composite plate (Fig.1) subjected to uniform tension σ_0 was examined. The dimensions of the plate are 45 mm wide, 2.45 mm thick, and 300 mm long. The material properties are E_{11}=70.24 Gpa, E_{22}=35.45 Gpa, G_{12}=11.4 Gpa, v_{12}=0.246, and $\varphi=45^o$ (Fig.2). The notch angle is γ, the angle between the notch centerline and the horizontal axis is β, and the specific notch length-to-plate width ratio a/W is 1/4 (a=11.25 mm). The SIFs of the notched plates for different γ and β are then calculated using the least-squares and H-integral methods. The results were shown in Table 1, which indicates that the two methods obtain very similar results, where the difference is smaller than 0.2%. This table also indicates that $F_I(\delta_2)$ and $F_{II}(\delta_1)$ are zero when the material is symmetric along the x-axis. Since the least-squares method only requires the data inside the problem, it is suitable to find notch SIFs for displacement-based experimental methods. In this study, the image correlation experiment was used to find the SIFs of the specimen with the composite plate mentioned above with γ= 60^{o} and β =30^{o}. The calculated SIFs are shown in Table 2, which indicates that the least-squares results compared with the H-integral are acceptably accurate.

Conclusion

This study finds the notch SIFs using a least-squares method with finite element analyses or displacement-based experiments. Numerical and experimental simulations show that the SIFs evaluated from the least-squares method are accurate. The major advantage of the proposed method is that the procedure is simple and systematic, so it can be applied to any finite element code or displacement-based experiment without difficulties.

References

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Table 1.Errors (Eq.4) and Notch SIFs of the composite plate as shown in Fig.1 using the least-squares method under plane stress finite element analysis with 8-node isoparametric elements

Case		H-integral method and FEM						
γ	β	δ_1+1	δ_2+1	$F_I(\delta_1)$	$F_{II}(\delta_1)$	$F_I(\delta_2)$	$F_{II}(\delta_2)$	Error(Eq.4)
30^{o}	15^{o}	0.5016164	0.5920702	1.1583425	0.1689095	0.0026248	0.0813925	0.07%
45^{o}	15^{o}	0.5058322	0.6553068	1.1779514	0.1499930	0.0094361	0.1346436	0.07%
60^{o}	15^{o}	0.5148517	0.7346341	1.2132533	0.1240083	0.0242860	0.2147028	0.08%
90^{o}	15^{o}	0.5556550	0.9414783	1.3506922	0.0643572	0.0591464	0.3801779	0.12%
30^{o}	45^{o}	0.5026805	0.6437079	0.8867402	0	0	0.6086726	0.15%
45^{o}	45^{o}	0.5083190	0.7239656	0.9026860	0	0	0.7034551	0.11%
60^{o}	45^{o}	0.5181135	0.8056867	0.9372923	0	0	0.7981899	0.04%
60^{o}	30^{o}	0.5173640	0.7848696	1.0682859	0.0432251	0.0520920	0.6004097	0.08%

Table 2. Error (Eq.4) and Notch SIFs of the composite plate as shown in Fig.1 using the image correlation experiment and least-squares method

Case		H-integral method and FEM				
γ	β	$F_I(\delta_1)$	$F_{II}(\delta_1)$	$F_I(\delta_2)$	$F_{II}(\delta_2)$	Error(Eq.4)
60^{o}	30^{o}	1.0806839	0.0437267	0.0588698	0.6785300	5.5%