ANLYTICAL SOLUTION TO ALL EDGE CLAMPED CYLINDRICAL PANELS WITH FOURIER SERIES SOLUTION FUNCTION

Humayun R.H. Kabir

Department of Civil Engineering, Kuwait University, P.O. Box 5969, Safat 13060, Kuwait
E-mail: kabir@civil.kuniv.edu

Analytical solutions of all edge clamped cylindrical panels are important boundary value problems to study, particularly with advanced fibre reinforced lamination. Its complex forms of coupled partial differential equations that represent the characteristic behaviours of panels have attracted many researchers (Librescu et al, Kabir) to study its in-depth behaviours. Two sets of solution functions in the form of double Fourier series both not satisfying the geometric boundary conditions have been selected. to solve five highly coupled partial differential equations in five unknowns that characterize a moderately thick shell behaviour. The analysts or designers use various shell theories (CST), for example, Love (1927), Donell (report no. 479) Flugge(1960), Sanders(1959), and Reissners (Bert and Kumar 1982), depending on the requirements, of shell geometries in their analyses or design works. The solution approach of shells/panels using such shell theories can be categorized into two broad groups: analytical and approximate numerical methods. Due to the complexity involved in the former one, the later one has become very popular in the last three to four decades. However, the users of the later method always desire to see the analytical-based solution to check the accuracy of their approximations. Normally, these are accomplished solving some bench-mark analytical solutions.

An extensive literature search has revealed that the aforementioned shell theories for the case of cylindrical panels with all edge clamped boundary conditions did receive analytical solutions to boundary value problems. The two double-Fourier series based solution approaches, Navier and Levy types, that are available in the literature for a century are applicable to very limited boundary conditions. For example, Naviers (Bert et al. 1982) approach is suitable for a SS3-type boundary conditions (Hoff and Rehfield 1965; Kabir and Chaudhuri 1993) at all edges, and Levy-type (Librescu et al. 1989) approach is suitable for a SS3-type boundary conditions at two opposite edges while other edges can be combination of SS1, SS2 or C4 boundary conditions.

No other boundary-value problems, e.g. C4 at all edges, are reported in the literature for the study of cylindrical panels. Only recently, Kabir and Chaudhuri (1994) have studied the static response of cylindrical panels utilizing a boundary discontinuous double Fourier series solution functions. In their study, they have developed a generalized solution approach of Naviers. The cylindrical panel behaviours are represented by five displacement unknowns such as $u_1, u_2, u_3, \theta_1, and\ \theta_2$. The solution functions they have assumed in the form of double Fourier series as

$$u_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right),$$

$$u_2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right),$$

$$u_3 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right),$$

$$\theta_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right)\ and$$

$$\theta_2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} E_{mn} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right)$$

The above assumed functions fully satisfy the geometric boundary conditions. Now the following displacement functions are assumed:

$$u_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} cos\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right),$$

$$u_2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} sin\left(\frac{m\pi x}{a}\right) cos\left(\frac{n\pi y}{b}\right),$$

$$u_3 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right),$$

$$\theta_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn} sin\left(\frac{m\pi x}{a}\right) cos\left(\frac{n\pi y}{b}\right)\ and$$

$$\theta_2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} E_{mn} cos\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right)$$

The above set of solution functions partially satisfy (except u_3) the geometric boundary conditions.

The partial differential equations based on Sanders' kinematic relations are as follows:

$$N_{1,1} + N_{6,2} + cM_{6,2} + \frac{Q_1}{R_1} = 0$$

$$N_{6,1} + N_{2,2} - cM_{6,1} = 0$$

$$Q_{1,1} + Q_{2,2} - \frac{N_1}{R_1} = q$$

$$M_{1,1} + M_{6,2} - Q_1 = 0$$

$$M_{6,1} + M_{2,2} - Q_2 = 0$$

Table 1 shows comparison of edge and central moment for cylindrical panels with respect to finite method. Average converged results are reported.

A comparison of eigen-values with the present method and finite element method for a cylindrical panel is listed in Table 1.

Moment	b/a = 1; r/a = 10; a/h = 50			b/a = 1; r/a = 10; a/h = 100		
	Analytical	FEM (ANSYS)/Analytical	FEM (NISA)/Analytical	Analytical	FEM (ANSYS)/Analytical	FEM (NISA)/Analytical
M	1	0.98	0.98	1.0	0.97	0.97
M(edge)	1	0.93	0.93	0.9	0.9	0.9

Table 1: Comparison of the present and finite element solutions.

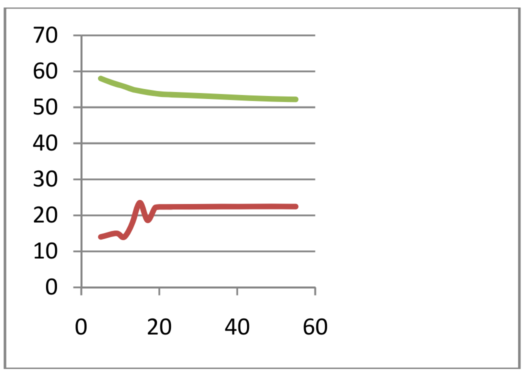

Fig. 1 Convergences of Moments for Isotropic Square Plate

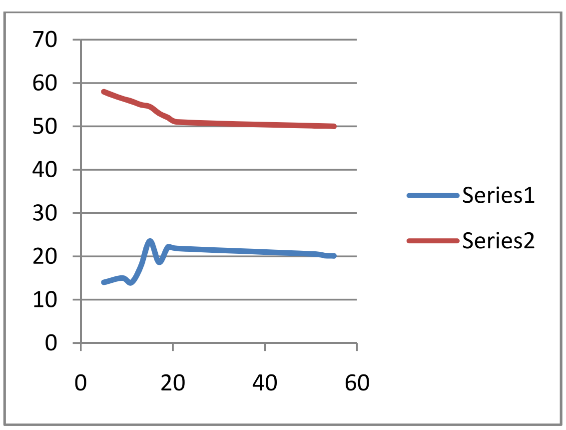

Figure 2 Convergences Moments for Isotropic Square Cylindrical Panel.

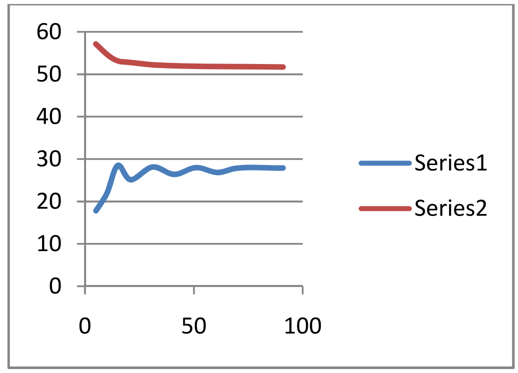

Figure 3 Convergences of Moments for a Cross-Ply (0/90) Cylindrical Panel.

CONCLUSION

An analytical solution is developed and compared with the commercially available finite element solution.

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