NEW CONTINUUM APPROACH IN DYNAMICS OF FRACTURE

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Introduction

Attempting to understand the phenomenon of dynamic material strength the main efforts of scientific community was focused on definition of functional strength parameters (dynamic material strength and dynamic fracture toughness) by analogy with quasistatic fracture theory. Nowadays, such approach looks unviable and inconvenient for engineers due to necessity to define material strength functions experimentally for every shape of loading pulse. And in 1988 Morozov, Petrov and Utkin have proposed a new criterion of crack initiation under high-rate loading [1] extended to describe the spalling phenomenon in 1990 [2]. It is based on the notion of incubation time – the characteristic time of microfracture (relaxation) processes anticipating the macrofracture event. The new incubation time approach turned out to be very powerful in the problems of fracture dynamics and many peculiarities of dynamic strength phenomenon was soon explained (see, e.g., [3]), including the nature of dynamic branch (appearing under extremely short loading pulses) of temporal dependence of material strength. The main advantage of incubation time criterion consists in using material constants (not functions of rate and so on but depending only on sample size and temperature) that can be defined from static experiments (see, e.g., [4]). These material parameters are the static material strength σ_C, the static fracture toughness K_{IC} and the incubation time of fracture τ (defining as time asymptote of dynamic branch of temporal dependence of strength).
Later, the incubation time criterion was extended to other dynamic transient processes. Among them we can mention the pulsed electrical breakdown in insulators [5], cavitation in liquids [5, 6], initiation of yielding [7] and melting under high-rate loading [8]. In listed problems the generalization of incubation time criterion was used, namely

$$\frac{1}{\tau}\int_{t-\tau}^{t} G_s^{\beta}(t')dt' \le G_C^{\beta}, G_s^{\beta}(t) = \operatorname{sign} G(t)\ G^{\beta}(t). \quad (1)$$

Here G is the dynamically changing function characterizing the intensity of external loading (local stress, pressure or stress intensity factor depending on physical process under consideration), G_C represents its critical value under "slow" (quasi-static) loading, β is some positive constant (it is usually obtained from specific experimental data) and τ is the incubation time (different for different physical phenomena) associated with the dynamics of relaxation processes preparing the structural transition in continuum.

Continual-Temporal Approach

So, a variety of different dynamic phenomena anticipated by relaxation-type processes obey the incubation time criterion which reveals the fundamental role played by incubation time regarding to abrupt structural changes in continuum. But incubation time criterion (1) allows an integral consideration of relaxation processes and does not provide their continual description at the microscale. Here we would like to present the kinetic description of abrupt structural changes in continuum (called *continual-temporal approach*) based on the notion of incubation time. Proposed approach operates with *the damage function* which can be considered in fracture problems as instant local microfracture state (this function describes the microfracture evolution including the processes of nucleation, interaction and following coalescence of microfracture – microcracks, microdamage and so on).
Let us consider a *spatially isotropic* process of microfracture evolution and fix an arbitrary small solid volume. Its mass is denoted as m, its volume before deformation is V_0, whereas the total volume of microfracture (damage) accumulated inside the chosen portion is V_*. Thus, during the damage accumulation process its volume changes as $V = V_0 + V_*$. The change of volume is obviously accompanied by variation of local density, described by the mass conservation law. Introducing the damage function $\theta = \frac{dV_*}{dV}$, under the most common assumptions on the form of expression for the divergence of local velocity of medium particles, the mass conservation law takes on form [4]

$$\frac{d\theta}{dt} = \frac{1}{\tau\zeta}\frac{G_s^{\beta}(t) - G_s^{\beta}(t-\tau)}{G_C^{\beta}}\theta^{\alpha_1}(1-\theta)^{\alpha_2}. \quad (2)$$

The way of definition of the damage function θ defines the range of it values: $\theta \in (-\infty, 1]$ and $\theta = 0$ corresponds to intact (defectless) material, the local state of macroscopic failure is referred to $\theta = 1$ whereas $\theta < 0$ can be treated as some "suppressed" state. Therefore, the initial condition for eq. (2) could be stated as

$$\theta(0) = 0\,, G(0) = 0 \text{ supposing } G(t \to 0+) \neq 0. \quad (3)$$

The criterion of macro-failure obviously has the form

$$\theta(t_*) = 1, \quad (4)$$

where t_* is the time to fracture (time from the moment of loading application till the moment of macroscopic fracture). The dimensionless parameters ζ, α_1 and α_2 in eq. (2) have to be defined by satisfying to known fracture criteria in particular cases. Thus, from correspondence of condition (4) to incubation time criterion (1) one could obtain

$$\zeta = \frac{\Gamma(2 - \alpha_1 - \alpha_2)}{\Gamma(1 - \alpha_1)\,\Gamma(1 - \alpha_2)}, \qquad 0 \leq \alpha_1, \alpha_2 < 1. \quad (5)$$

From the other side, parameters α_1 and α_2 could be determined from analysis of long-term quasistatic fracture processes. Indeed, eq. (2) with fracture criterion (4) gives for the creepage problem (considering the tension of uniform bar by constant external load P – the specific load on the unit of initial cross-sectional area) [4]:

$$\frac{d\theta}{dt} = \frac{\beta}{\zeta \sigma_C^\beta} P^\beta (1 - \theta)^{-\zeta - \beta}, \;\; \alpha_1 = 0, \alpha_2 = 1 - \zeta. \quad (6)$$

And eq. (6) coincides exactly with Rabotnov's creepage equation [9]

$$\frac{d\theta}{dt} = BP^k (1 - \theta)^{-r-k} \text{ if } B = \frac{1}{\zeta \sigma_C^\beta}, \beta = k, \zeta = r \quad (7)$$

Applying eq. (2) to analysis of fatigue crack growth under cyclic loading we will obtain the dependence of crack growth rate on the stress intensity range ΔK_I in the form [4]

$$\frac{\Delta a}{\Delta N} = \frac{2}{\pi} \left(\frac{K_{IC}}{\sigma_C}\right)^2 \left(\frac{\Delta K_I}{K_{IC}}\right)^\beta, \;\; \alpha_1 = 1 - \zeta, \alpha_2 = 0. \quad (8)$$

Comparison of eq. (8) with Paris equation [10]

$$\frac{\Delta a}{\Delta N} = C_p (\Delta K_I)^{m_p}, \quad (9)$$

commonly used for description of fatigue crack propagation, yields

$$C_p = \frac{2}{\pi} \frac{K_{IC}^{2-\beta}}{\sigma_C^2} \quad \text{and} \quad m_p = \beta. \quad (10)$$

Results and Discussion

So, we have constructed the kinetic equation describing the evolution of microfracture during incubation processes anticipating the macroscopic fracture. Proposed continual temporal approach gives classical fracture criteria both in static and dynamic cases as well as the equations of Rabotnov (for creepage) and Paris (for fatigue cracks). By the way, we have obtained the remarkable result in damage and fatigue mechanics – the analytical relations between the parameters of correspondent equations. Indeed, eqs. (7) and (10) yield

$$B = \frac{1}{r \sigma_C^k} \quad \text{and} \quad C_p = \frac{2}{\pi} \frac{K_{IC}^{2-m_p}}{\sigma_C^2}. \quad (11)$$

Therefore, we have shown that phenomena of static, long-term and dynamic strength of materials can be described from the common positions by eq. (2).

Application of proposed approach to the cleavage problem (the most interesting phenomenon of dynamic fracture because of changing of stress's sign during loading process) one can find in [4].

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