Modeling on Constitutive Behavior for Short Fiber Reinforced Composites

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Introduction

Short fiber reinforced composites have several attractive characteristics that make them worthy of consideration for other applications. Therefore, short fiber reinforced composite materials have been extensively investigated because they are more economical and impact resistant [1]. One of the earliest attempts to explain the reinforcing effect of short fibers was described by Cox and is now referred to as the shear lag theory [2,3]. The predicted modulus value obtained by Cox model is significantly smaller than the experimentally observed values in the short fiber composites. In fact, the Cox model gives a underestimation of the strength due to the neglect of stress transfer across the fiber ends [4,5].

During decades, several ways of accounting fiber end stresses have been proposed. Taya considered that the fiber end stress is treated as the average stress of remote matrix though it has the limitation for no stress intensification between fibers [6]. Their results also show an underestimation for the prediction of elastic modulus in short fiber reinforced composites. There have been more attempts such as Clyne model considering that the fiber end stress is assumed to the mean value between the average value in the matrix remote from the interface and the peak value in the fiber in the absence of fiber end stresses [4]. It has been shown that Clyne model is an improvement over the Cox model or Taya model in terms of the prediction of Young's modulus data of MMCs (Metal Matrix Composites) such as SiC-Al composites.

Recently, Starink also proposed within the framework of shear lag model, assuming that the fiber end stress would approximate to the mean between the average value in the matrix remote from the interface and the peak value in the fiber in the presence of fiber end stresses [5]. More recently, Kim reported the fiber end stress with stress concentration phenomenon for the accurate prediction of reinforcement effects based on the material property [7]. Therefore, an analytical approach of reinforcement effects in short fiber reinforced composites is abbreviated to investigate the estimation of elastic modulus and constitutive relations.

Model

In the present model, the comparatively short fiber is considered as aligned and uniaxial stress applied in the axial direction of the fibers. Using equilibrium conditions as reported in the previous paper [7], the governing is obtained as Eq. (1).

$$\frac{d^2\sigma_f}{dz^2} = \frac{n^2}{r_f^2}(\sigma_f - E_f\varepsilon_c) \tag{1}$$

where ε_c is the far-field composite strain and n is the dimensionless parameter as Eq. (2).

$$n^2 = \frac{2E_m}{E_f(1+v_m)\ln(P_f/V_f)} \tag{2}$$

Here, v_m is Poison's ratio of the matrix, $s(=l/r_f)$ is fiber aspect ratio for hexagonal arrangement of fiber. V_f and V_m are volume fractions of the fiber and the matrix, respectively. Then, above equation admits the solution as shown below. Eq. (1) has the solution of Eq. (3) called Cox model.

$$\sigma_f^{Cox} = E_f\varepsilon_c\left\{1 - \frac{\cosh(nz/r_f)}{\cosh(ns)}\right\} \tag{3}$$

Consequently, E_c^{Cox} is obtained as below.

$$E_c^{Cox} = V_f E_f\left(1 - \frac{\tanh(ns)}{ns}\right) + V_m E_m \tag{4}$$

In the meantime, Taya and Arsenault reported the fiber axial stress by taking fiber end stress into account without stress concentration as below [6].

$$E_c^{Taya} = V_f E_f \left\{ 1 + \left(\frac{E_m}{E_f} - 1 \right) \frac{\tanh(ns)}{ns} \right\} + V_m E_m \quad (5)$$

Clyne postulated that σ_e would approximate to the mean between the average value in the matrix remote from the interface and the peak value in the fiber in the absence of fiber end stresses as below [4].

$$E_c^{Clyne} = V_f E_f \left\{ 1 + \left(\frac{E^{Clyne}}{E_f} - 1 \right) \frac{\tanh(ns)}{ns} \right\} + V_m E_m \quad (6)$$

where E^{Clyne} is $E^{Clyne} = \frac{1}{2}[E_f\{1 - \sec h(ns)\} + E_m]$.

Recently, Starink proposed within the framework of shear lag model, σ_e would approximate to the mean between the average value in the matrix remote from the interface and the peak value in the fiber in the presence of fiber end stresses as below [5].

$$E_c^{Starink} = V_f E_f \left[1 + \left(\frac{E_m / E_f - 1}{2 - \sec h(ns)} \right) \frac{\tanh(ns)}{ns} \right] + V_m E_m \quad (7)$$

More Recently, Kim reported a closed form solution of composite mechanics as below [9].

$$E_c^{Kim} = V_f E_f \left\{ 1 + \left(\sqrt{\frac{E_m}{E_f}} - 1 \right) \frac{\tanh(ns)}{ns} \right\} + V_m E_m \quad (8)$$

Results and Discussion

The results of present study are compared with experimental data as well as the prediction of other models for various fiber aspect ratios, fiber volume fractions, and fiber/matrix Young's modulus ratios. For the numerical calculation, typical elastic moduli of materials are chosen as E_f=450GPa for the fiber and E_m=71GPa and 78GPa for the matrix. Fig. 8 shows the predicted and measured composite/matrix Young's modulus ratio as a function of fiber aspect ratio for V_f=30% in SiC-Al composites. In Fig. 1, it is found that ROM model fits a continuous fiber composite presumably and that Halpin-Tsai model overestimates from experimental data whereas Cox model and Taya model underestimate from experimental data. However, it can be seen that Clyne model and Starink model are below than experimental data for the very small fiber aspect ratio though they give well agreement with Kim model as the fiber aspect ratio increases. Presumably, all seven models converge to the ROM model as the fiber aspect ratios increases.

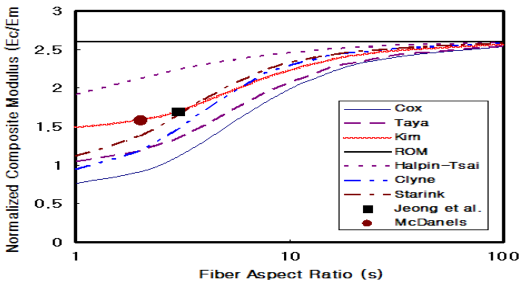

Fig. 1 Predicted and measured composite/matrix Young's modulus ratio as a function of fiber aspect ratio for V_f=30% in SiC-Al composites.

Conclusions

A closed form solution of the shear lag theory has been derived and evaluated for the prediction of elastic modulus in a short fiber reinforced discontinuous composite materials. The accuracy and relative simplicity of the present model have been exploited to derive an analytical model for the stress transfer in a composite and have also been compared with other theories. The predictions of present model to predict composite Young's modulus are fairly consistent with the measurements of SiC-Al MMCs.

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