POST-BUCKLING OF CORRUGATED PAPERBOARD

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Introduction

Corrugated paperboard is most commonly used for packaging applications. The detrimental effects of loads leading to buckling, on the condition of packaging and consequently of packaged goods, gives justification for study of the deformation behaviour of corrugated paperboard in buckling.

Published analytical results modelling post-buckling behaviour of corrugated paperboard have been found to show discrepancy with experimental results in [1]. The present study has extended from the author's previous work in reproducing the corrugated paperboard panel buckling model of [1], which employed a semi-energy method and accounts for initial geometric imperfection. Several different approaches were used to model buckling of corrugated paperboard panel in the attempt to identify the reasons for the discrepancy and improve agreement between published experimental results and the analytical results obtained. Post-buckling behaviour is modelled in this study since the effect of plate curvature in increasing stiffness and hence allowing further loads to be sustained beyond the first buckling load, is taken into account.

Experimental

Materials

The dimensions and material properties assigned to three-layered corrugated paperboard panel models were taken from the model of a single wall corrugated paperboard panel considered by Nordstrand [1]. A 0.4 m square panel was modelled (refer to Fig. 1).

The profile of the corrugated paperboard was taken to be the C-flute, with a take-up factor, α, of 1.45. The take-up factor, α is the ratio of the length of paper in the corrugated layer to the length of the facings.

The models consider corrugated paperboard as an orthotropic single lamina with material properties equivalent to the overall material properties of corrugated paperboard. An initial imperfection with an amplitude of out-of-plane displacement, A_o of 0.0008 m the average experimental value in [1] was used.

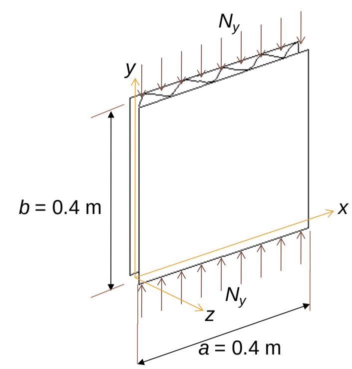

Fig. 1 Three-layered corrugated paperboard panel with uniform in-plane load distribution, N_y.

Apparatus and Procedures

The loading condition modelled was of a panel with uniformly distributed loading in the *y*-direction, N_y (refer to Fig. 1). The condition of plane stress was assumed. The boundary conditions for the models in this study include zero out-of plane displacement, constant in-plane normal displacement and zero in-plane shear along edges of the panel, and free rotation about the panel edges. These boundary conditions differ from the model in [1], where the unloaded edges were free to displacement in-plane.

The analytical models of the post-buckling behaviour of corrugated paperboard panels, in terms of the resultant applied load, *P* and the out-of-plane displacement amplitude, *A*, were created using MATLAB.

The analytical approaches used in this paper for modelling buckling of corrugated paperboard are the semi-energy method [2] and Galerkin's method. Both analytical models used a double Fourier series for the out-of-plane displacement function, *w*, as in the model of post-buckling of in-plane loaded plates in [3]. In both approaches, the Airy stress function, F used was of similar form to the functions used in [4] and [3] for modelling vibration and buckling of in-plane loaded plates using the Galerkin method. In both approaches, the compatibility equation in [1], relating the stress function, F to the out-of-plane displacement, w, was used to determine the constants in the stress function, F.

In both methods, the Newton-Raphson method was used to solve a system of non-linear equations; with products of the undetermined coefficients of out-of-plane displacement amplitude, A.

Results and Discussion

Single and two-term solutions showing the relationship between the applied load and the out-of-plane displacement for the corrugated paperboard panel were obtained using the semi-energy approach and Galerkin's method, with Airy stress functions from [3].

The single-term solution using the Galerkin's method is shown in Fig. 2. The two-term Galerkin's method solution and single and two-term semi-energy solutions have not been included here as there is some uncertainty in the validity of the post-buckling load vs. displacement plots of these solutions. The single-term Galerkin's method gives a load-displacement plot which is similar to the published analytical result.

Critical buckling loads obtained for the single-term solutions using the two approaches are shown in Table 1. The lowest critical buckling loads in the two-term solutions for both approaches are the same as those of the single-term solutions. Comparisons of critical buckling loads obtained to those in [1], show both approaches having slightly lower buckling loads compared to the analytical results in [1], which was 958 N. The experimental critical load in [1] is 814 N. Possible sources of discrepancy between the analytical and the experimental results in [1] are that the experimental boundary conditions, particularly in-plane conditions, not being exactly as modeled and the material modeling of the corrugated paperboard being less than ideal. The reasons for the discrepancy between obtained and published analytical results are under investigation.

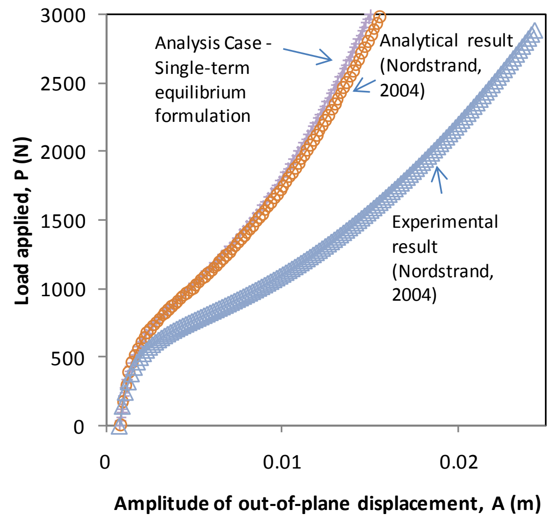

Fig. 2 Comparison of applied load vs. amplitude of out-of-plane displacement from models for post-buckling of corrugated panel.

Table 1 Comparison of critical buckling loads.

Results	Critical load, *P crit* (N)
Single-term minimisation of energy	944
Single-term equilibrium formulation	939

Conclusion

The post-buckling behavior of a corrugated paperboard panel was modeled analytically using a semi-energy and Galerkin's methods. The critical buckling loads obtained were slightly lower than the analytical published results but higher than the experimental published result. The load vs. displacement plot of the single-term Galerkin's method solution showed good agreement with the published analytical result.

References

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