Simulation of FRP Thin-Walled Beams by a Combined Analytical and Finite Element Beam Model

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Introduction

To analyze FRP thin-walled beams such as wind turbine blades, the finite element method (FEM) is the most commonly used method in recent years. However, FEM is inefficient based on the numerous calculations required. Moreover, FEM doesn't directly sense the global mechanical behaviors like the analytical method does. Due to these two reasons, we hope to find another way to achieve the analysis in the preliminary design and optimization of blades.

Hence, this study uses a mixed method, the Combined Analytical and Finite element Beam model (hereafter called CAFB method) with an inclination angle correction to simulate FRP thin-walled beams. The mixed method uses the analytical method in deriving the section stiffness matrix of each section. And, each section is coupled by the finite element beam models. In this way, the beams can preserve all structural mechanical properties while reducing the model's degree of freedoms and the computational time. Moreover, the CAFB method can also help us understand the behaviors of beams. Wherein, the inclination angle correction is to correct the overestimated axial stiffness due to the topology of beams.

Methodology

$$\begin{bmatrix} N \\ M_y \\ M_z \\ T \\ M_\omega \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ & k_{22} & k_{23} & k_{24} & k_{25} \\ & & k_{33} & k_{34} & k_{35} \\ & & & k_{44} & k_{45} \\ sym. & & & & k_{55} \end{bmatrix} \begin{bmatrix} U_{,x} \\ \beta_{y,x} \\ \beta_{z,x} \\ \phi_{,x} \\ \phi_{,xx} \end{bmatrix} \quad (1)$$

Many section analytical methods have been carried out in different considerations. The used terms in force-displacement relationship of these methods may be distinct. One method by Jung et al [1,2] is adopted in this study, which is based on a uniform section assumption. The relationship by Jung contains 7 forces and the corresponding deformations including extension, bending, torsion, warping torsion, and transverse shear deformations. In this study, the transverse shears are not considered; the relationship is shown in Eq. (1). However, the section analytical method can't directly approach the target blade as Fig. 1 because it is not a uniform beam. Therefore, to simply adapt to the non-prismatic geometry as blade, a modification on the material properties to correct the inclination angles in blade models is also used in this study.

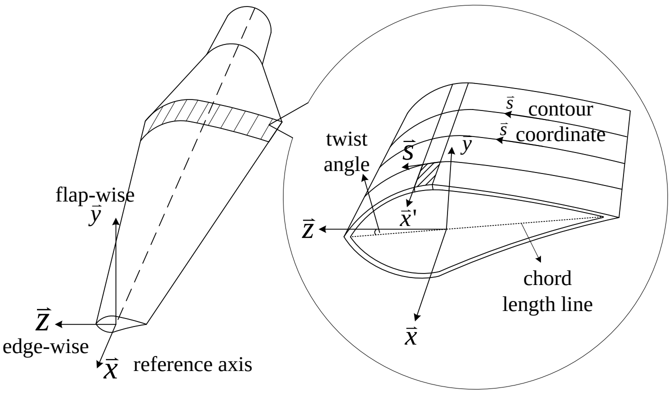

Fig. 1: Geometry, profile and the coordinate system.

The blade can be seen as several discrete beam elements, whose beam stiffness matrix can be derived by virtual work equation. Subsequently, by combining the beam stiffness matrices, the nodal displacements come out from Eq. (2).

$$\{\Delta\} = [K]^{-1}\{F\} \quad (2)$$

Where the blade stiffness matrix $[K]$ is established by beam stiffness matrices, and $\{F\}$ is the force vector. The stress and strain distributions may also be evaluated by the displacement vector $\{\Delta\}$.

Comparison

(i) Comparison about computation time

The number of used elements immensely affects the computing efficiency. It is observed the discernible differences in computation time between CAFB and FEM in Table 1. This comparison only includes the inverse matrix procedure as Eq. (2) because it usually takes the largest portion of computation time in whole calculation. To get the inverse matrix, LU decomposition method is usually used; its order of computation is proportional to the third power of degrees of freedom. In this way, the computation time can be reduced several orders of magnitude by CAFB instead of FEM.

Table 1. Comparisons

Method	Elements	Nodes	DOFS	Computation time
CAFB	13	14	14×7	$\propto (14\times7)^3$
FEM (shell)	1750	1818	1818×6	$\propto (1818\times6)^3$
Ratio	0.74%	0.77%	0.90%	0.000073%

(ii) The influence of the inclination angle

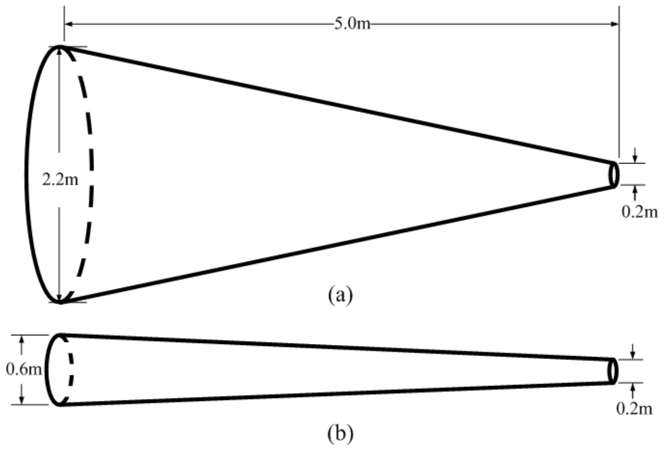

Fig. 2: Geometry of model (a) and model (b).

For the reason to understand the influence of the inclination angle, two models as shown in Fig. 2 with different levels of inclination angle (11.31° & 2.29°) are calculated in bending. Although the dimension of model (a) is not standard, it is still seen as a beam model for the purpose to know the effectiveness of the correction for large inclination angles. Table 2 shows the tip displacement difference, which is compared to FEM. It reveals the correction effectively improve the accuracy.

Table 2 : Relative differences (%) between CAFB and FEM

Ply	Model (a)		Model (b)	
	Without Correction	With Correction	Without Correction	With Correction
$[0_3]$	-24.57	**3.00**	-4.39	**-2.90**
$[15_3]$	-16.42	**-0.04**	-2.31	**-1.49**
$[30_3]$	-9.48	**-2.74**	-0.76	**-0.45**
$[45_3]$	-7.63	**-4.13**	-0.34	**-0.18**
$[60_3]$	-8.43	**-5.02**	-0.56	**-0.41**
$[75_3]$	-10.58	**-5.63**	-1.18	**-0.96**
$[90_3]$	-11.86	**-5.80**	-1.47	**-1.20**

(iii) Real Blade Geometry

Here, a real blade geometry, developed in [3], is also taken into calculation. Some large inclination angles exist in the transition region between airfoil and root of the blade; these angle are ranged about 10 degrees. By applying a unit load in flap-wise and edge-wise direction, respectively, the relative differences are shown in Table 3. These cases appear the same phenomenon with models (a) and (b); large relative difference occurs if the fiber orientation angle ranges from 0 to 15 degrees without correction. By the inclination angle correction, the relative difference decrease in all cases, especially, at small fiber orientation angles.

Table 3 : Relative differences (%) between CAFB and FEM

Ply	Flap-wise		Edge-wise	
	Without Correction	With Correction	Without Correction	With Correction
$[0_6]$	-12.70	**2.82**	-24.67	**-1.93**
$[15_6]$	-6.78	**2.18**	-14.70	**-0.83**
$[30_6]$	-3.01	**0.60**	-5.69	**0.23**
$[45_6]$	-2.28	**-0.38**	-2.91	**0.31**
$[60_6]$	-2.46	**-0.51**	-2.66	**0.69**
$[75_6]$	-3.46	**-0.49**	-4.34	**0.75**
$[90_6]$	-4.10	**-0.40**	-5.93	**0.36**

Conclusion

This study presented a combined analytical and finite element beam model with an inclination angle correction to analyze FRP thin-walled, blade-like beams such as wind turbine blades. By using a simple correction to adjust the material stiffness, the prismatic analytical method can be used to analyze blade-like beams. The correction can assist CAFB method in getting a more accurate result in the presented cases especially at small fiber orientation angles (0°~15°). Compared to FEM, the CAFB method is more efficient in calculation. It is roughly estimated that by the usage of CAFB instead, the computation time can be reduced several orders of magnitude. And, the relative differences in the tip displacements of a blade model in this study are less than 3%.

References

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