On the Governing Equations for the Vibration Problem of the Transversely Isotropic Magneto-Electro-Elastic Rectangular Plate

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Introduction

In these years, the so-called magneto-electro-elastic (MEE) composite has been vigorously discussed due to its multiphase properties including elasticity, electricity and magnetism as well as the interactions among them. Several valuable research results determining either the material coefficients or the dynamical characteristic of various structures about MEE material have been achieved widely from multidisciplinary fields, see Ref. [1]-[6]. More recently, Liu and Chang [7] proposed a rather compact form governing the transverse vibration of a MEE plate by adopting the thin plate theory, and the exact solution for the natural frequency of a bi-layered $BaTiO_3$ - $CoFe_2O_4$ composite is presented in their study. In this paper, the governing equations for the vibration problem of a MEE rectangular plate are derived based on the Classical Laminate Plate Theory instead.

Formulations

Constitutive equations:

$$\sigma_{ij} = C_{ijkl} S_{kl} - e_{kij} E_k - q_{kij} H_k , \qquad (1)$$

$$D_i = e_{ikl} S_{kl} + \varepsilon_{ik} E_k + g_{ik} H_k , \qquad (2)$$

$$B_i = q_{ikl} S_{kl} + g_{ik} E_k + \mu_{ik} H_k , \qquad (3)$$

Extended strain-displacement relations:

$$S_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right), \quad E_i = -\phi_{,i} , \quad H_i = -\psi_{,i} \qquad (4)$$

Equations of motion and Maxwell theories:

$$\sigma_{ij,j} + f_i = \rho \frac{\partial^2 U_i}{\partial t^2} \qquad (5)$$

$$D_{j,j} - f_e = 0 \qquad (6)$$

$$B_{j,j} - f_m = 0 \qquad (7)$$

Classical Laminate Plate Theory (CLPT):

(1) Straight line perpendicular to the midsurface (i.e. transverse normals) remains straight after deformation.

(2) The transverse normals do not experience elongation (i.e. they are inextensible).

(3) The transverse normals rotates such that they remain perpendicular to the midsurface after deformation.

(4) Only the transverse magnetic field E_3 is considered, i.e. $E_1 = E_2 = 0$. ([8])

(5) Only the transverse magnetic field H_3 is considered, i.e. $H_1 = H_2 = 0$. ([7])

Kinematics for the CLPT:

$$U_1 \equiv U(x,y,z) = u(x,y) + z\beta_1(x,y) , \qquad (8)$$

$$U_2 \equiv V(x,y,z) = v(x,y) + z\beta_2(x,y) , \qquad (9)$$

$$U_3 \equiv W(x,y,z) = w(x,y) , \qquad (10)$$

where $\beta_1 = -\frac{\partial w}{\partial x}$ and $\beta_2 = -\frac{\partial w}{\partial y}$.

Equilibrium equations:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^2}{\partial t^2}\left(\frac{\partial w}{\partial x}\right) \qquad (11)$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\frac{\partial^2 v}{\partial t^2}-I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial w}{\partial x}\right) \quad (12)$$

$$\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+\bar{p}_3 = I_0\frac{\partial^2 w}{\partial t^2}-I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) . \quad (13)$$

Derived from Liu and Chang [7]:

$$\frac{\partial \phi}{\partial z}=-\frac{\Delta_1}{\Delta}z\nabla^2 w+\phi_0(x,y) \quad (14)$$

$$\frac{\partial \psi}{\partial z}=-\frac{\Delta_2}{\Delta}z\nabla^2 w+\psi_0(x,y) \quad (15)$$

Governing Equations can be derived

$$A_{11}\frac{\partial^2 u}{\partial x^2}+(A_{12}+\frac{1}{2}A_{66})\frac{\partial^2 v}{\partial x\partial y}+\frac{1}{2}A_{66}\frac{\partial^2 u}{\partial y^2} +E_{31}\frac{\partial \phi_0}{\partial x}+Q_{31}\frac{\partial \psi_0}{\partial x}=\rho_P h\frac{\partial^2 u}{\partial t^2} \quad (16)$$

$$\frac{1}{2}A_{66}\frac{\partial^2 v}{\partial x^2}+(\frac{1}{2}A_{66}+A_{12})\frac{\partial^2 u}{\partial x\partial y}+A_{11}\frac{\partial^2 v}{\partial y^2} +E_{31}\frac{\partial \phi_0}{\partial x}+Q_{31}\frac{\partial \psi_0}{\partial x}=\rho_P h\frac{\partial^2 v}{\partial t^2} \quad (17)$$

$$-D_{11}\frac{\partial^4 w}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}-D_{11}\frac{\partial^4 w}{\partial y^2} -G_{31}\frac{\Delta_1}{\Delta}\frac{\partial \phi_0}{\partial x}\nabla^4 w-S_{31}\frac{\Delta_2}{\Delta}\frac{\partial \psi_0}{\partial x}\nabla^4 w+\bar{p}_3 =\rho_P h\frac{\partial^2 w}{\partial t^2}-I_2\left(\frac{\partial^2 \ddot{w}}{\partial x^2}+\frac{\partial^2 \ddot{w}}{\partial y^2}\right) \quad (18)$$

where

$$\{A_{ij},B_{ij},D_{ij}\}=\int_{-h/2}^{h/2}(1,x_3,x_3^2)C_{ij}d\ x_3,$$

$$\{E_{ij},F_{ij},G_{ij}\}=\int_{-h/2}^{h/2}(1,x_3,x_3^2)e_{ij}d\ x_3,$$

$$\{Q_{ij},R_{ij},S_{ij}\}=\int_{-h/2}^{h/2}(1,x_3,x_3^2)q_{ij}d\ x_3,$$

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