Buckling Analysis of Corrugated Shipping Containers

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Introduction

Corrugated board materials (CB) are essentially sandwich structures consisting of two flat plates (*liners*) separated by a sine wave-shaped medium (*flute*) whose tips are glued to the *liners* (Fig. 1). Because CB materials play a major role in the packaging economy, they need to be strong in compression in order to withstand the weight of other packages stacked on top during warehousing and transportation. Determination of the compression strength would allow manufacturers to choose the correct grade of the corrugated board. The production of paperboard gives rise to three mutually perpendicular directions: the machine direction (MD), the cross machine direction (CD), and the thickness (*calliper*) direction (ZD). As a result, buckling equations for the *orthotropic* plate theory can be used. The box is modeled as four simply-supported *orthotropic* plates joined together at right angles. When loaded under compression, the box would fail as soon as the maximum load-induced stress component in any plate exceeds the critical buckling stress of the material.

Formulation

Uniform compression of a thin *orthotropic* plate subjected to a compressive load per unit length, N_y, (Fig. 1), is governed by [1]:

$$D_{11}w_{,xxxx}+2Hw_{,xxyy}+D_{22}w_{,yyyy}+N_y w_{,yy}=0 \quad (1)$$

where

$$D_{11}=E_1 t^3/12(1-v_{12}v_{21}) \quad (2)$$

$$D_{22}=E_2 t^3/12(1-v_{12}v_{21}) \quad (3)$$

$$D_{33}=G_{12}t^3/12 \quad (4)$$

$$D_{12}=v_{12}D_{11}, \quad H=D_{12}+2D_{33} \quad (5)$$

The subscript *comma* in (1) denotes partial differentiation w.r.t. the variables after the *comma*, and t is the CB *calliper*. The “1-2" directions are essentially the “x-y” (or “MD-CD”) directions. The following empirical equation was used to determine G_{12} [2]:

$$G_{12}=0.387\sqrt{E_1E_2} \quad (6)$$

where E_1 and E_2 are the elastic moduli in the MD, and CD directions, respectively. These moduli related to the flexural stiffness components according to Eqs. (2) & (3).

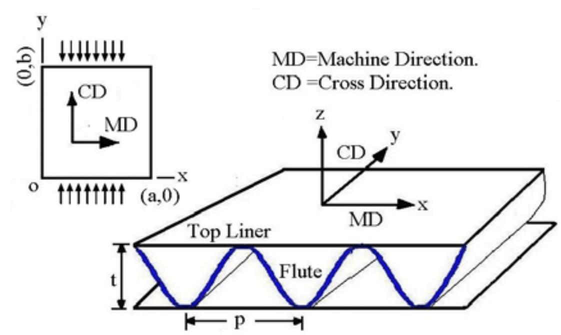

Fig. 1- Corrugated Board (CB) and the Orthotropic Plate Model.

For a simply-supported rectangular (axb) plate, assuming that the plate buckles into one-half wave, the following function satisfies the related boundary conditions [1,3,4]:

$$w(x,y)=A\sin(\pi x/a)\sin(\pi x/b) \quad (7)$$

where A is the amplitude.

Substitution of (7) in the stress equations [1] gives the critical stress [1],

$$\sigma_{cr}=\frac{\pi^2}{a^2t}(D_{11}r^2+2H+D_{22}/r^2) \quad (8)$$

where r=b/a.

In a typical box of dimensions LxWxh with L>W, the critical stress is related to the L-h plate (b/a=h/L). Once the critical stress is determined, the buckling load can be computed from

$$F_{cr}=2(L+W)t\,\sigma_{cr} \qquad (9)$$

Examples

Consider a box, 15 (in.)x 12 (in.) x 10 (in.), made of the 200-series CB with the C-flute having the following material properties [5]:

Liner thickness =0.009 in.
Flute height =0.125 in.
Calliper t=0.143 in.
Pitch p=0.325 in.

Using these parameters the method was applied to 16 different cases of CB having different $(D_{11},D_{22})=(D_x,D_y)$ values [5]. The resulting critical buckling loads were computed from (9). Because the Poisson ratio of CB is not well defined, considered $v_{12}=0.34$ [6], and varied $v_{12,}$ $v_{21}=k_v v_{12}$. For different values of k_v ranging from 0.5 to 1 the buckling loads remained stable and consistent. Sample results are shown in Table 1.

Comparison with the McKee Equations:

Empirical equations to estimate the top-to-bottom compression strength of corrugated boxes subjected to a concentrated load applied at the center of the top plate have been developed by McKee [7]:

$$F_o = 2.028\ f_L^{0.746}\ (D_x D_y)^{0.127}\ P^{0.492} \qquad (10)$$

$$F_o = 5.874\ f_L t_0^{0.508}\ P^{0.492} \qquad (11)$$

where P is the box perimeter (in.) and f_L (lbf/in.) is the Edge Crush Test (ECT) of the CB which is determined experimentally [5]. Equation (10) and its short-form (Eq. 11) were used to assess the validity of the results.

Conclusions

A convenient method for determination of the buckling load in a box made of corrugated materials has been presented. The method is based on the buckling equation of the thin *orthotropic* plate. The method is straightforward and gives consistent results for the buckling load of a CB of arbitrary shape and material properties. Additionally, the method does not require determination of the ECT load which is required in Eqs. (10) and (11).

References

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Table 1. Results for F_{cr}(lbf), $v_{12}=v_{21}=0.34$

Statistical Results	Plate Theo.	McKee	
		Eq. (10)	Eq. (11)
F_{cr}(Min)	503.2	573.6	530.8
F_{cr}(Max)	833.0	852.2	814.1
F_{cr}(Mean)	681.2	749.4	686.6
F_{cr}(st. dev.)	77.3	69.1	69.4