DYNAMIC ANALYSIS WITH STRESS RECOVERY FOR FUNCTIONALLY GRADED MATERIALS: NUMERICAL SIMULATION AND EXPERIMENTAL TESTING

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Introduction

In this work it was attempted to design and build material specimens whose behavior, under mechanical loading, is similar to that of functionally graded materials; to perform dynamic vibration tests on the produced specimens; and to numerically simulate the vibration tests in order to validate a proposed numerical model for functionally graded material characterization. Fraga[1] and Romero et al.[4] present the extent of this work in greater detail.

Materials and Methods

The problem was divided in the following phases: (i) the production of the specimens; (ii) the preparation and execution of the vibration laboratory tests; (iii) the mathematical modeling for the analytical resolution of the composite beam vibration problem; (iv) the numerical simulation of the dynamic tests; and (v) the comparative analysis between analytical, experimental, and numerical results.

The specimens were built by adhesive bonding of metal strips, with constant thickness along their length. The width of Specimen 1, a 2.00mm thick strip of AISI 1020 Steel and a 1.20mm thick strip of Brass, was 19.60mm. Specimen 2, with a 3.40mm strip of Aluminum and a 1.70mm strip of Copper, was 19.66mm wide. Specimen 3, which was 19.80mm wide, was composed of two strips of AISI 1020 Steel (1.50mm and 1.80mm) separated by a 1.50mm strip of Copper. The useful length was 275.00mm for all specimens. Each specimen was fixed as a cantilever beam using a standard mechanical vibrations lab kit. For each test an accelerometer was fixed at one of the four predetermined positions along the beam. The vibration displacements were processed through Fast Fourier Transform to produce natural frequency results. These results were corrected to account for the weight and position of the accelerometer.

The composite beam strength of materials model was used to develop the analytical solution for each specimen.

A finite element software for the dynamic behaviour of functionally graded materials was developed. This software uses generalized isoparametric elements for material properties interpolation and a local macroelement residual stress recovery technique to recover stress values, as well as the α method of Hilber-Hughes-Taylor for time stepping[2].

The new post-processing technique [3] involves solving a local residual problem at a macroelement which encompasses all elements adjacent to a node. The initial value problem is stated by:

$$\nabla \cdot \sigma + b - \rho \ddot{u} = 0, \quad (1)$$

with the initial conditions given by:

$$\left\{ \begin{array}{rcl} \mathbf{u}(x,0) & = & u_0, \ \ in \ \ \Omega, \\ \dot{\mathbf{u}}(x,0) & = & \dot{u}_0, \ \ in \ \ \Omega, \\ \mathbf{u} & = & u_0, \ \ on \ \ \Gamma_0, \\ \sigma \cdot \mathbf{n} & = & t_n, \ \ on \ \ \Gamma_1. \end{array} \right. \quad (2)$$

The residual equations are given by:

$$\left. \begin{array}{rcl} R_1 & = & \sigma - CBd, \\ R_2 & = & \nabla \cdot \sigma + b - \rho \ddot{u}, \\ R_3 & = & (\sigma \cdot \mathbf{n} - t_n). \ on \ \Gamma_1 \end{array} \right\} \quad (3)$$

The solution of this variational problem produces the smooth recovered stress values.

Results and discussion

Table 1 presents natural frequency results for the first vibration mode, obtained by statistical analysis, for specimen 01, the adjusted results for the undamped case, and the correction of the results (due to the acceleration sensor). The results are shown for different positions of the accelerometer. Similar results were obtained for the other specimens.

Table 1: Mean Natural Frequencies for Specimen 01 - 1^{st} Vibration Mode

Pos. (mm)	Damped (Hz)	Undamped (Hz)	Corrected (Hz)
40	29.1125	29.1144	29.1150
100	28.1802	28.1829	28.3248
160	25.8840	25.8906	27.7885
220	22.7705	22.9066	30.3693

Table 2 represents the natural frequencies obtained by FFT after the finite element displacement results, computed with different numbers of simulation steps.

Table 2: Natural Frequencies (Hz) obtained by numerical simulation - 1^{st} Vibration Mode (Δt = 1.0×10^{-3} s).

Steps	**Specimen**		
	01	**02**	**03**
50	39.7465	39.7465	61.6359
100	30.5300	50.1152	50.1152
300	32.8341	46.6590	50.1152
500	31.6820	47.8111	52.4194
700	32.8341	47.8111	51.2673
1000	32.8341	47.8111	51.2673
1200	32.8341	47.8111	51.2673

Analysing Table 2, a tendency towards stabilization of the natural vibration frequencies for each specimen can be observed, after a minimum number of simulation steps. These stabilized natural frequencies were taken as representative of the movement.

The natural vibration frequencies obtained by different approaches (experimental, analytical, and numerical simulation) are shown in Table 3.

Table 3: Natural Frequencies (Hz) for the 1^{st} Vibration Mode - Obtained by Different Approaches

Specimen	**Experimental**	**Analytical**	**Numerical**
1	28.8994	29.7899	32.8341
2	44.2015	46.6047	47.8111
3	44.4137	50.4471	51.2673

Analytical and experimental results have shown a maximum percentual difference of 13.6% for Specimen 3 (the sandwich beam). The differences between analytical and experimental results were deemed acceptable, considering the natural experimental variation and the existence of thin adhesive layers between adjacent metal laminae (the adhesive layer was not considered in the composite beam model used).

The computed difference between experimental (validated by analytical results) and numerical results was at its maximum 15.4%, for Specimen 3. This difference allows the validation of the numerical simulation considering the theoretical and experimental approximations taken.

The maximum percent difference between numerical and analytical results was 10.2% for specimen 1. For specimens 2 and 3 the differences were 2.6% and 1.6%, respectively. Convergence between numerical and analytical results was observed.

Final remarks

The agreement between numerical and analytical results is remarkable. This result is quite promising, since the analytical modeling of real functionally graded materials (with continously varying structure and mechanical properties) is quite cumbersome and time consuming.

Final results can be taken as a validation of the numerical model for dynamic analysis of functionally graded materials. The frequency analysis has shown convergence of the proposed method with the increase in the number of simulation steps.

References

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