

MECHANICAL STRESS OF FUNCTIONALLY GRADED COMPOSITE PLATE

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Introduction

Functionally graded materials (FGMs) are gradually changing non-homogeneous material from one surface to the other, and typically are made up of ceramics and light metals. Three-dimensional thermo-mechanical deformations of a simply supported FGM rectangular plate using an asymptotic method was analyzed [1]. Also, three-dimensional thermo-mechanical buckling analysis of FGM plates using finite element method was performed [2]. In this work, an 18-node solid element was used, and the assumed strain mixed formulation was applied.

Present study performs the stress analysis of FGM composite plates using 3-D finite element method with 18-node solid element. In the FGM layer, material properties are assumed to be varied continuously in the thickness direction according to a simple power law distribution in terms of the volume fraction of mixture. In addition to this, the effective material properties are obtained by the linear rule of mixtures. Tensile stress and the compressive stress according to the FGM thickness ratio and volume fraction distribution are analyzed in detail.

Modeling of FGM composite plates

A FGM composite plate, composed of ceramic, FGM, and metal layers, of length a , width b , and thickness h is described in Ref.[2]. In the figure, h_c , h_m and h_f indicate the thicknesses of ceramic, metal and FGM layers, respectively. In the FGM region, material properties are assumed to be varied in the thickness direction only and the bright and dark areas correspond to ceramic and metal particles, respectively. Ceramic and metal layers are assumed to be homogeneous and isotropic. The thickness ratios of ceramic, metal, and FGM layers are denoted by r_c , r_m and r_f

$$r_f = \frac{h_f}{h}, \quad r_c = \frac{h_c}{h} = r_m = \frac{h_m}{h} = \frac{1}{2}(1 - r_f)$$

Numerical results and discussions

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The stress analysis of all clamped square Si3N4-SUS304 FGM composite plate is performed. Material properties are listed in Ref.[1]. To evaluate the stress distribution, the tensile stress and the compressive stress are investigated for the sinusoidal load over the top surface of the plate. According to the linear rule of mixtures [1], the tensile strength σ_{Bt} and compressive strength σ_{Bc} of the model at each point are assumed as

$$\sigma_{Bt}(\bar{z}) = \sigma_{Btm} V_m(\bar{z}) + \sigma_{Btc} V_c(\bar{z})$$

$$\sigma_{Bc}(\bar{z}) = \sigma_{Bcm} V_m(\bar{z}) + \sigma_{Bcc} V_c(\bar{z})$$

Fig. 1 depicts the tensile and compressive strengths along the thickness direction. When z/h is increased, the compressive strength increases, however, the tensile strength decreases in FGM layer. So as to evaluate the stress according to the strength, the stress ratio σ^* is introduced using the tensile stress ratio $\bar{\sigma}_t$ and the compressive stress ratio $\bar{\sigma}_c$

$$\sigma^* = \begin{cases} \bar{\sigma}_t = \sigma_{xx} / \sigma_{Bt} & \sigma_{xx} \geq 0 \\ \bar{\sigma}_c = \sigma_{xx} / \sigma_{Bc} & \sigma_{xx} \leq 0 \end{cases}$$

To avoid failure, the condition $|\sigma^*| < 1$ should be fulfilled, and when $|\sigma^*|$ becomes small, the structure achieves better stress reduction.

The maximum tensile stress $(\bar{\sigma}_t)_{\max}$ and the maximum compressive stress $|\bar{\sigma}_c|_{\max}$ of FGM composite plates according to FGM thickness ratio and volume fraction index is shown in Table 1. In FGM composite plates, as the volume fraction index is increased, the tensile stress decreases. When the volume fraction index n is increased, the compressive stress decreases. This is because the contained quantity of ceramic increases as the volume fraction index is increased. When $n \geq 1$, the compressive stress decreases as the FGM thickness ratio r_f is increased. From Table 1, in overall cases, the tensile stresses have larger values than the compressive stresses. Thus, the tensile stress is the most important factor for evaluation of maximum stress ratio for the FGM composite plates under mechanical load.

Fig. 2 describes through-the-thickness distributions of the stress ratio σ^* according to the thickness ratio. Through-the-thickness distributions of the stress ratio σ^* with various volume fraction indexes are depicted in Fig. 3. Numerical data of the models for figures are $a/h=50$ and $\bar{q}=5$.

Conclusions

The stress analysis considering tensile and compressive stresses are investigated for Functionally Graded composite plates under mechanical load. 3-D finite element method is adopted and an 18-node solid element is used for more accurate modeling of material properties in the thickness direction. Material properties are assumed to be varied continuously in the thickness direction according to a simple power law distribution. The stress analysis for various volume fraction ratios are performed for Si3N4-SUS304 FGM composite plates.

References

1. Reddy, J.N. and Cheng, Z.Q. Three-dimensional thermo-mechanical deformations of functionally graded rectangular plates. *Eur. J. Mech. A/Solids.*, **20** (2001) 841-855.
2. Na, K.S. and Kim, J.H. Three-dimensional thermo-mechanical buckling analysis for functionally graded composite plates. *Compos. Struct.* 73 (2006) 413-422.

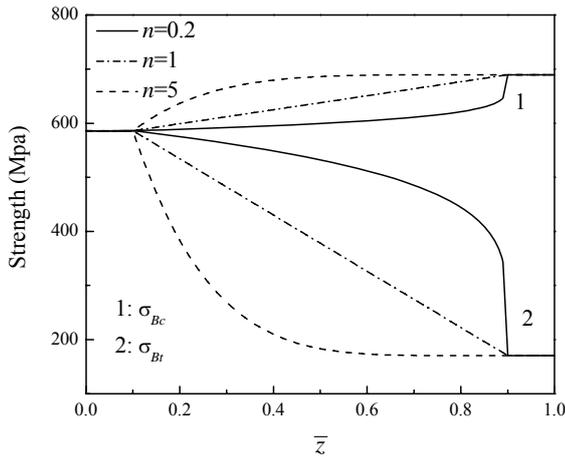


Fig. 1 Tensile strength and compressive strength along the thickness direction ($r_f=0.8$).

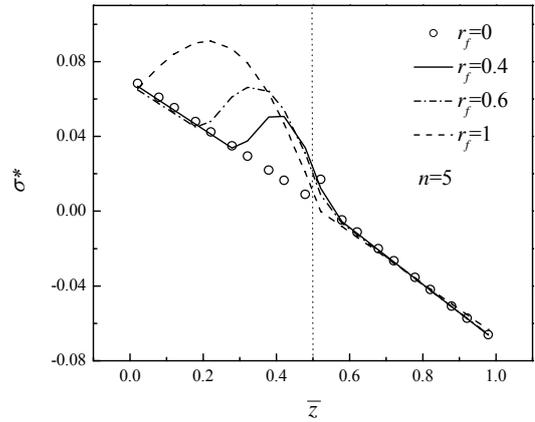


Fig. 2 Through-the-thickness distribution of the stress ratio σ^* with respect to the FGM thickness ratio.

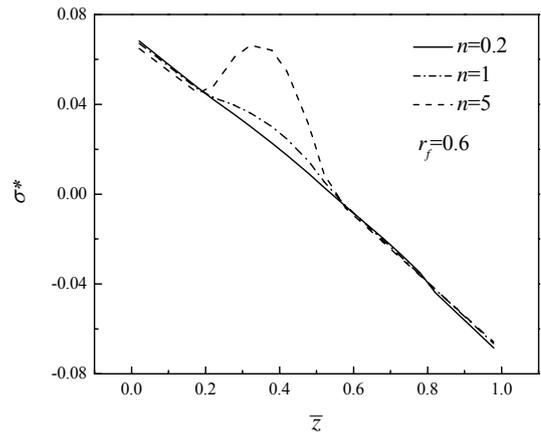


Fig. 3 Through-the-thickness distribution of the stress ratio σ^* with the variation of volume fraction index.

Table 1 Maximum stresses of FGM composite plate for FGM thickness ratio and volume fraction index.

$n \setminus r_f$	$(\bar{\sigma}_t)_{\max}$		$ \bar{\sigma}_c _{\max}$	
	0.4	0.6	0.4	0.6
0.2	0.4618	0.4736	0.1093	0.1094
0.5	0.4591	0.4649	0.1090	0.1085
1	0.4574	0.4590	0.1085	0.1075
2	0.4566	0.4557	0.1078	0.1060
5	0.4563	0.4535	0.1066	0.1035