

Application of Dimensional Analysis to Predict Poly Ethylene Oxide (PEO) Fiber Diameters from Electrospinning Process

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Introduction

Dimensional analysis was used to develop a functional relation between the fiber diameter and related process parameters for PEO electrospinning process. To investigate this approach, data¹ from more than two dozen researchers over a decade (1999-2008) from more than a dozen institutions was compiled and analyzed for aqueous PEO (Poly Ethylene Oxide) solutions. This data was used to validate the effectiveness of the proposed dimensional analysis. In this approach a number of dimensionless parameters were developed from first principles and they were used to develop a functional relationship that can be used in real time in a manufacturing environment of PEO solutions.

Identifying Non-dimensional Parameters

Parameters chosen for dimensional analysis included fiber radius (R_f in m), nozzle radius (R_n in m), viscosity (μ in Kg/(m-sec)), conductivity (K in $\text{Sec} \cdot \text{C}^2 / (\text{Kg} \cdot \text{m}^3)$), concentration (C in Kg/Kg), flow rate (Q in m^3/Sec), and electric field (E_0 in $\text{Kg} \cdot \text{m} / \text{sec}^2 \cdot \text{C}$). In addition, following identities were used in this dimensional analysis:

$$Q = R_n^2 v_n = R_j^2 v_j \dots\dots\dots (1)$$

$$R_f / R_j = C \dots\dots\dots (2)$$

Suffixes n and j were values at the nozzle (n) and jet (j) at the point of fiber formation.

Sixteen non-dimensional parameters were developed using consistent SI units. It was noted that a few of these parameters were well known, namely, Reynolds Number, Peclet Number, Weber Number, Ohnesorge Number, and Froude's Number (15). Additionally, NEF (Non-dimensional Electric Field)^{2,3} was also

used in this investigation that was introduced by Helgeson et al^{2,3}.

By plotting these sixteen non-dimensional parameters against non-dimensional fiber diameter (R_f / R_n), it was found that there were no meaningful correlation with most of these parameters except for Reynolds Number, Peclet Number, and NEF mentioned above. These three non-dimensional parameters included fluid properties (density, viscosity, conductivity, and concentration), electric field parameters (applied field and permittivity), and flow parameters (instantaneous velocity and corresponding jet radius). Using Equations (1) & (2) instantaneous jet velocity and corresponding radius were expressed in terms of flow rate (Q) and concentration (C). A new parameter called Non-dimensional Flow Concentration Rate (NFCR) was introduced in this investigation to manipulate the Reynolds Number in terms of flow rate, Q , rather than jet velocity, v_j . It may be noted that NEF and NFCR represented non-dimensional parameters for electric and flow fields.

Results & Discussions

Fig 1 showed the plot of \ln of NFCR against \ln of NEF. Surprising linearity between natural log terms of NFCR and NEF was simplified further to get the following equation

$$R_f = A \cdot C \cdot K \cdot Q / E_0^2 \dots\dots\dots (3)$$

Where A is proportionality constant.

This was done by simplifying the linear \ln - \ln relation of Fig 1. It was argued that the constant A in Equation (3) was an experimental constant that depended on polymer molecular weight of solute, solvent characteristics, permittivity of the environment, etc. For a typical experimental or

manufacturing environment, these parameters were typically constant. This observation was used to calculate the value of A for each of these experiments using experimental data of fiber diameter and corresponding process parameters.

Fig 1. Correlation between NFCR and NEF

Averaging the value of A value, Equation (3) was used to re-calculate the predicted value of fiber diameters using the same Equation. Fig 2 showed the correlation between predicted and experimental fiber diameters. To highlight the excellent agreement, Ideal Data (same value for predicted and experimental data) was included in the graph. Constant A in these calculations was used as the average value obtained from the data and Equation (3) above. Considering the wide range of data from such diverse groups, the proposed approach deemed powerful.

Fig. 2 Comparison of Experimental and Predicted Diameters of PEO Nanofiber

In real manufacturing environment, A can be estimated using few initial data points. Once it is estimated, this should not change in a typical batch environment. Equation (3) then can be used to control fiber diameter by manipulating any of the process parameters of this equation, namely, flow rate, concentration, or applied electric field.

Conclusions:

Dimensional analysis was used to predict the quantitative roles of various process parameters of ambient electrospinning process of aqueous PEO solution. A simple predictive relation for fiber diameter was developed in terms of conductivity, concentration, flow rate, and electric field. Further investigation will be continued to validate the potential of this method for other polymer systems. This method, if successful, can become a powerful tool to control fiber diameters in real time in other electrospinning processes.

Acknowledgement:

This paper is dedicated in loving memory of Dr. Hashim Mahdi who was the driving force behind this work.

References

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