

# THEORETICAL PREDICTION OF WORK-OF-FRACTURE OF CARBON/CARBON COMPOSITES

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## Introduction

Fiber reinforcement is an effective method to improve fracture toughness of ceramics, and therefore many researchers have focused on it over 30 years. In these studies, interfacial debonding or fiber pullout was investigated with many experiments; however, quantitative prediction of the resultant toughness has not been fully discussed. Sutcu [1] was the first to show theoretical background of the total dissipation energy during fracture of fiber-reinforced ceramics in 1988. His insight was right on target of this subject, by pointing out that strength distribution of fiber controlled pullout length, however, it was clearly irrelevant that the dissipation energy of interfacial debonding was equivalent to fracture energy of matrix. The present authors [2] therefore developed a theory to predict the total dissipation energy of fiber-reinforced ceramics composites by combining energetics of microfracture process and a fracture location theory. In this presentation, the theory is verified with experimental data of work-of-fracture of carbon/carbon composites.

## Theory

### *Energetics of microfracture processes*

Consider that a single fiber is embedded in semi-infinite matrix, and the applied tensile stress  $\sigma_{max}$  is given to the fiber. As shown in Fig.1, at

some stress, interfacial debonding occurs, and then its debonding length gradually increases with increasing the applied stress. Since the debonded part of fiber is automatically pulled out from the matrix, interfacial frictional energy is dissipated by this process. Owing to limited space, we just show the final equation relating to the dissipation energy  $U_F^{debonding}$  of a single fiber subjected to the applied stress  $\sigma_{max}$  as below,

$$U_F^{debonding} = \frac{2\pi\tau^2}{3E_f} \left[ \left( \frac{r\sigma_{max}}{2\tau} \right)^3 - 3(x^\Delta)^2 \left( \frac{r\sigma_{max}}{2\tau} \right) + 2(x^\Delta)^3 \right] \quad (1)$$

,where  $E_f$  is Young's modulus of fiber,  $\tau$  is interfacial frictional stress,  $x^\Delta$  is a constant relating to material parameters,  $r$  is radius of fiber.

With increased the applied stress further, the fiber breaks at some depth  $x$  from the surface. The depth  $x$  is determined by strength distribution of fiber. After fiber breaking, the fiber is gradually pulled

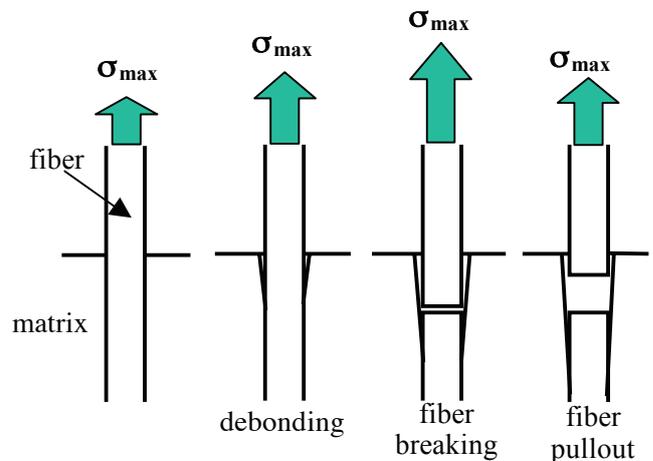


Fig.1 Fracture processes of fiber-reinforced ceramics composites.

out from the matrix until the fiber is completely apart from the surface. The dissipation energy  $U_F^{\text{pullout}}$  of a single fiber broken at the depth  $x$  can be expressed as below,

$$U_F^{\text{pullout}} = \pi r \tau x^2 \quad (2)$$

#### Joint probability density function

Assume that a tensile test is conducted under displacement rate control for a notched specimen of unidirectional fiber-reinforced ceramics. The fracture mode is limited to single matrix fracture, and the fracture origin of fiber is only assigned to surface defect. The fracture strength of fiber can be expressed 2-parameteric Weibull distribution function. Under these assumption, the joint probability density function  $h(\sigma_{\max}, x)$  can be expressed in the following equation,

$$h(\sigma_{\max}, x) = \exp \left\{ - \left( \frac{S_e}{S_0} \right) \left( \frac{\sigma_{\max}}{\sigma_0} \right)^m \right\} \frac{\partial}{\partial \sigma_{\max}} \frac{1}{S_0} \left( \frac{\sigma_{\max}}{\sigma_0} \right)^m \left( 1 - \frac{2\tau}{r\sigma_{\max}} x \right)^m \quad (3)$$

,where  $m$  and  $\sigma_0$  are shape and scale parameters of Weibull distribution,  $S_0$  is the unit surface area,  $S_e$  is the effective surface area.

#### Total dissipation energy

The dissipation energy by interfacial debonding is obtained as below,

$$W_{\text{debonding}} = A \cdot \left( \frac{r\sigma_0}{2\tau} \right)^3 \frac{\Gamma\left(\frac{m+4}{m+1}\right)}{\left(\eta\sigma_0\right)^{\frac{1}{m+1}}} - 3A \cdot (x^\Delta)^2 \left( \frac{r\sigma_0}{2\tau} \right) \frac{\Gamma\left(\frac{m+2}{m+1}\right)}{\left(\eta\sigma_0\right)^{\frac{1}{m+1}}} + 2A \cdot (x^\Delta)^3 \Gamma(1) \quad (4)$$

,where  $A$  and  $\eta$  are parameters depending on material.  $\Gamma(*)$  is the gamma function. Similarly, the dissipation energy by fiber pullout is obtained as below

$$W_{\text{pullout}} = \frac{2B}{(m+1)(m+2)} \left( \frac{r\sigma_0}{2\tau} \right)^2 \frac{\Gamma\left(\frac{m+3}{m+1}\right)}{\left(\eta\sigma_0\right)^{\frac{2}{m+1}}} \quad (5)$$

,where  $B$  is also a parameter depending on material. When we add  $W_{\text{debonding}}$  and  $W_{\text{pullout}}$ , and then divide

it by the factor 2, we obtain the theoretical expression of work-of-fracture frequently used in ceramics composites to show their toughness.

Cabon fiber / furan resin-based carbon composites were made by changing the heat treatment temperature up to 3000°C, and their work-of fracture values were measured by single edge notched beam method. Fig.2 shows comparison between the experimental data (○) and theoretical prediction (●). It can be seen good agreement between the two, and therefore the theory is verified with experimental data.

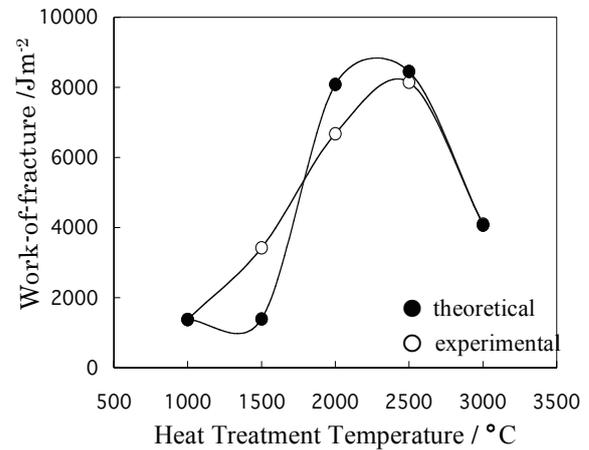


Fig.2 Work-of-fracture of C/C composites.

#### References

1. Sutcu, M. Statistical fiber and single crack behavior in uni-axially reinforced ceramics composites. *J.Mater.Sci.*, **23** (1988) 928-933.
2. Yasuda, K. Onuma, T. Matsuo, Y. and Shiota, T. Evaluation of dissipation energies during debonding and fiber-pullout processes of a C/C composite, *Proceedings of the 20th International Japan-Korea Seminar on Ceramics, Nov.20-22, Matsue, Japan*, **20**, (2003) 367-370.