The Non-linear Damping Coefficient in the Contact Model of Discrete Element Method

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Introduction

The non-linear damping coefficient in the contact model of the discrete element method (DEM) has attracted growing attention in the literature in the last decades. Zhang and Whiten [1] examined different methods for DEM simulation that are used to calculate the forces and found that the commonly used linear model (Eq. (1)) gives unrealistic behaviors. The non-linear model (Eq. (2)) proposed by Tsuji *et al.* [2] based on the Hertzian contact theory predicts that impacting force starts from zero, and results are reasonable which are much closer to the experimental results than that from the linear model.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0 \qquad (1)$$

$$m\ddot{x}(t) + a\sqrt{mk}\,x(t)^{1/4}\,\dot{x}(t) + kx(t)^{3/2} = 0 \qquad (2)$$

However, for most contact models commonly used in DEM calculation, the impacting force and displacement return to zero at different time in a contact period. As shown in Fig.1, the non-linear model by Tsuji *et al.* predicts that the magnitude of impacting force increases to a maximum from zero, and reduces zero, and then changes sign at time $t_{f=0}$ and drops very rapidly back to zero at time $t_{x=0}$.

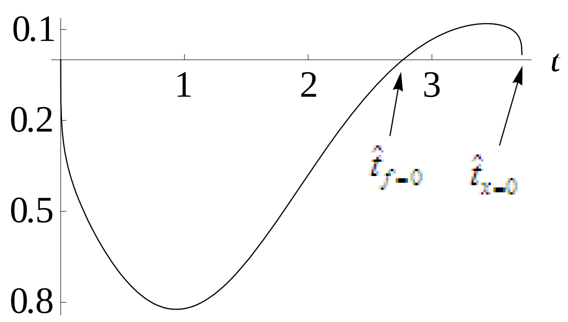

Fig. 1. Normalized force vs. normalized time using non-linear equation

Most works [3, 4] assumed that the particles separate at the time $t_{x=0}$, when the distance between the center of the colliding particles equals to the sum of the radii. But it is not realistic, because the points of times of $t_{x=0}$ and $t_{f=0}$ are different, and between time period $t_{x=0}$ and $t_{f=0}$, attractive force exists. So it is necessary to find a better form of the damping term to make that the impacting force and the displacement return to zero at the same time in a contact period.

The non-linear damping coefficient

It is found that the damping term cause the time $t_{x=0}$ and $t_{f=0}$ depart. Hunt and Crossley [5] derived the damping term $\lambda x(t)^p\,\dot{x}(t)^q$, where commonly the exponent q equals to one, and the coefficient λ and q are unidentified. So the damping term in Eq. (1) and (2) is the specific form of this expression.

We replace the damping term in the non-linear contact model of Tsuji *et al.*with $\lambda x(t)^p\,\dot{x}(t)$ and get a new non-linear contact model based on the Hertzian contact theory. The equation is as follows:

$$m\ddot{x}(t) + \lambda x(t)^p\,\dot{x}(t) + kx(t)^{3/2} = 0 \qquad (3)$$

where λ is $\alpha m v_0 \left(k/m v_0^{\,2}\right)^{2(1+p)/5}$, and v_0 represents the initial velocity.

To make each term non-dimensional, replace displacement x and time t respectively with:

$$x = \left(m v_0^{\,2}/k\right)^{2/5}\hat{x}$$

$$t = \left[\left(m v_0^{\,2}/k\right)^{2/5}\Big/ v_0\right]\hat{t}$$

Eq. (3) can be rewritten as:

$$\ddot{\hat{x}}(\hat{t}) + \alpha x(t)^p\,\dot{x}(t) + x(t)^{3/2} = 0 \qquad (4)$$

The initial conditions for the equation can be set to

$$\hat{x}(\hat{t})\big|_{\hat{t}=0} = 0,\quad \dot{\hat{x}}(\hat{t})\big|_{\hat{t}=0} = \dot{x}(t)\big|_{t=0}/v_0 = 1.$$

In the paper of Hunt and Crossley [5], when the velocity of initial collision is small, the exponent p in the damping coefficient should be identical with the non-linear exponent in the elastic force. So in this DEM contact model based on Hertz contact theory, the exponent of the damping coefficient should select 3/2.

Put the value of p into Eq. (3) and Eq. (4), we will obtain:

$$m\ddot{x}(t) + \lambda x(t)^{3/2}\,\dot{x}(t) + kx(t)^{3/2} = 0 \qquad (5)$$

$$\ddot{\hat{x}}(\hat{t}) + \alpha x(t)^{3/2}\,\dot{x}(t) + x(t)^{3/2} = 0 \qquad (6)$$

According to the initial conditions, we can make numerical differentiation calculations with respect to Eq. (6), and get the relation between the parameter α in the damping coefficient and the restitution coefficient e.

The approximate relation expression between the parameter α and the restitution coefficient e by numerical fitting is expressed by:

$$\alpha = -\ln(e)\frac{b}{c + \ln(e)} \qquad (7)$$

where the constants b and c are b= 6.66264, c=3.85238, respectively, and the domain of the equation is $\alpha \in [0, 20]$, or $e \in [0.05, 1.0]$.

Results and Discussion

The parameter α in the damping coefficient

According to the initial conditions, and setting the value range of the parameter α from 0 to 20, we can get the relation between the parameter α in the damping coefficient and the restitution coefficient e by numerical calculations with Eq. (6), shown as the solid line in Fig. 2.

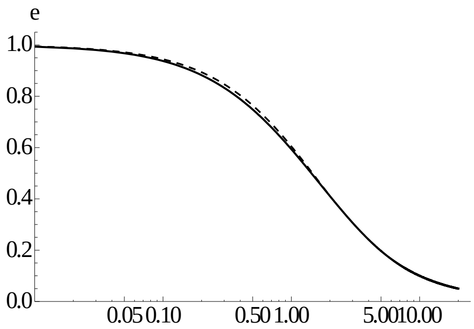

Fig.2. The relation between the parameter α and the coefficient of restitution e.

The fitting dash curve (shown in Fig. 2) corresponding to Eq. (7) is close to the numerical solution. But to make sure the value calculated by Eq. (7) is accurate enough, α should be selected as small as possible.

When the coefficient of restitution is given, the value of parameter α can be determined from Fig. 3 or Eq. (7), and the damping coefficient λ in Eq. (5) is expressed as:

$$\lambda = \alpha k / v_0 \quad .$$

The contact force

We solve Eq. (6) with different values of α, and plot the contact force-time curves corresponding with the values of α.

Fig. 3 shows when the value of exponent p equals to 3/2 and no matter how much the damping parameter α is, there will be no problem of attractive force as shown in Fig. 1, which means the contact force and the displacement will reach zero at the same time.

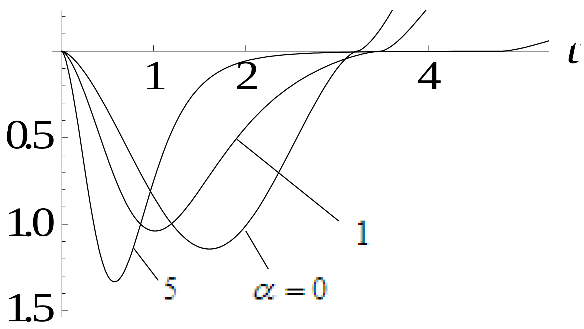

Fig. 3 Normalized force vs normalized time using non-linear Eq. (6)

Fig. 3 also shows when α is large enough (take the curve with α =5 for example) and the value of the restitution coefficient e is small ($e \approx 0.197$), the effect of the damping is apparent and the contact force approximates to zero in a short time. This resembles the condition of critical damping and over-damping in first-order vibration system, which also causes another question here: when the contact force increases to a certain number, it decreases quickly and approximates to zero.

Conclusions

It is very important to identify a reasonable damping model in the contact model for DEM simulation.

According to the damping term $\lambda x(t)^p \dot{x}(t)^q$ derived by Hunt and Crossley, we get a new damping term: $\lambda x(t)^{3/2} \dot{x}(t)$, where $\lambda = \alpha k / v_0$.

It is realistic to apply this damping term to the contact model in the DEM simulation. This contact model does not have the problem of attractive force as shown in Fig.1, which means the contact force and the displacement reach zero at the same time in this contact model. Through the further analysis, we deduce the expression between the restitution coefficient e and the parameter α in the damping term, which is $\alpha = -b \ln(e) / (c + \ln(e))$, where b=6.66264, c=3.85238.

We have to note that all the analysis above is based on theoretic study. For further research in the accuracy of the damping term in the contact mode in the DEM, more experimental analysis is needed.

Acknowledgements

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References

1. Zhang D and Whiten, WJ, The calculation of contact forces between particles using spring and damping models. Powder Technology, 1996, 88:59.
2. Tsuji, Y, Tanaka, T and Ishida, T, Lagrangian numerical simulation of plug flow of cohesionless particles in a horizontal pipe. Powder Technology, 1992, 71:239.
3. Malone KF and Xu, BH, Determination of contact parameters for discrete element method simulations of granular systems. Particuology, 2008, 6: 521-528.
4. Mishra, BK and Murty, CVR, On the determination of contact parameters for realistic DEM simulations of ball mills. Powder Technology, 2001, 115(3): 290–297.
5. Hunt, KH and Crossley, FRE, Coefficient of restitution interpreted as damping in vibroimpact. Journal of Applied Mechanics, 1975, 440-445.