Bone regeneration model (a new phenomenon in bone grafting)

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Introduction

Mathematical models are essential for refining laboratory/clinical experimental data since such experimental observations are subject to time and managerial constraints. The coincidental discovery of osteobstruction – a new phenomenon in bone regeneration assay during a sequential SPECT, histological and histomorphometric analysis on animal model in the validation of the Ogunsalu sandwich bone regeneration technique **[1]** is one of such assay that demand mathematical attention. Mathematical biology/tissue engineering has attracted desirable attention in the literature **[2,3,4].** The assumption is that bone or tissue can only be generated if the concentration of the growth factor exceeds a certain threshold value **[4]**. Models in the bone regeneration mechanism which incorporate cell Phenomena are in **[5].** In this paper we formulate mathematical models for the Ogunsalu osteobstruction phenomenon.

2. Materials and Method

After the observation of the osteobstructive phenomenon/mechanism and its cumulative SPECT, histological and histomorphometric evaluation in **[1]**, we utilized Fisher's equation **[4]** to speculate the bone cell concentration and other parameters. We assume that the sandwich membrane bag of the Ogunsalu D-GTR is cubic with unit sides, and the dimension of the S-GTR is a unit side square membrane.

3. **Mathematical models for osteobstruction**

3.1 ***Single GTR model***

The area of membrane is $L \times L = L^2$ unit square; osteobstruction parameter λ_1 unit square is the measure of soft tissue layer at graft site; effect of membrane/tissue interaction is ξ (real number). We take $D_1 = -D$; where D is the coefficient of diffusivity of the standard Fisher's equation. The relation between the rate of bone cell proliferation R_1 at graft site and bone cell proliferation R at graft site of Fisher's equation is modeled as

$$R_1 = (1 + \xi\lambda_1)R/(1 - \lambda_1) \ . \qquad (1)$$

The retardation effect on bone cell proliferation is obtained by ensuring that

$$(1 + \xi\lambda_1)/(1 - \lambda_1) > 0 \ \Rightarrow \ \xi < -\tfrac{1}{\lambda_1} . \qquad (2)$$

We define the general osteobstruction parameter in the two states of GTR by the relation

$$\lambda_m = surface\ area\ of\ membrane\ for\ GTR - 3m ;$$

$$m = 1,2 . \qquad (3)$$

i.e. for $L = 1$, $\lambda_1 = L^2 - 3 = -2$; $\Rightarrow \ \xi < \tfrac{1}{2}$. (4)

3.2 ***Double GTR model***

In this case for $L = 1$, $\lambda_2 = 6L^2 - 6 = 0$; the value of osteobstruction parameter is zero when $m = 2$ in equation (3). So $D_2 = D$ and $R_2 = R$.

4. Results

The governing mathematical equation for the S-GTR model is

$$B_{,t} = \nabla \cdot (-D\nabla B) + \left(\tfrac{1-2\xi}{3}\right)R(1 - B/B_*)B \ , \qquad (5)$$

where B is the bone cell concentration; the first term on the right hand side of equation (5) represents bone cell motility (migration) and the second term is the bone cell proliferation at graft site. While the equation governing the D-GTR model is

$$B_{,t} = \nabla \cdot (D_2\nabla B) + R_2(1 - B/B_*)B + \in \lambda_\circ , \qquad (6)$$

where $\in \lambda_{\circ}$ represents the shock of resorption of membrane and some biochemical reactions.

5. Discussions/conclusions

The significance of GTR (guided tissue regeration) membrane is basically to confine the bone graft to graft site. Obviously the thickness/type of membrane influences the osteoblastic activities and osseointegration of the graft material at the recipient site. We note interpretation of the model results visa vise, from 1st week to 11th week there are more osteoblastic activities at S-GTR site than at D-GTR site. Specifically in the 1st week of experiment the S-GTR site enabled very high concentration of bone cell which allowed higher rate of osteoblastic activities, as the bone graft is not enclosed in a membrane sack; thus $R_1 > R_2 = R$. i.e., equation (2) implies

$$(1+\xi\lambda_1)/(1-\lambda_1) > 1, \qquad (7)$$

hence very high rate of bone cell proliferation is attained. In later weeks to 11th week there is the formation of soft tissue wall (foreign body reaction) at S-GTR site which triggered reversal of bone cell migration from S-GTR site; i.e. the negative sign of diffusivity coefficient in (5). This reversal of influx of bone cell to S-GTR site led to retardation of osteoblastic activities at the site; compare with equation (6) where bone graft is enclosed in a membrane sack, there is no rapid osteoblastic activity within the 1st week into 11th week that could warrant accelerated osteoblastic activities at D-GTR site, thus there is a slow and steady flux of bone cell into D-GTR site which enabled slow and steady osteoblastic activities. Illustratively see fig.1; table1.

References

1. Ogunsalu C , Double-Guided Bone Regeneration using the Ogunsalu sandwich regeneration technique: single photon emission computerized Tomography, Histology, and Histomorphometric analysis. The University of the West Indies St. Augustine. PhD thesis 2009.

2. Murray J D. Mathematical biology, Vol. I, 3rd ed. New York, NY Springer 2002.

3. Murray J D. Mathematical biology, Vol. II, 3rd ed. New York, NY Springer 2003.

4. Sengers B G, Please C P, Oreffo R O C. Experimental characterization and computational modeling of two-dimensional cell spreading for skeletal regeneration. J.R. Soc. Interface 2007, **4**: 1107-1117.

5. Sherrat J A, Murray J D. Mathematical analysis of a basic model for epidermal wound healing. J. Math. Biology 1991, **29**: 389-404.

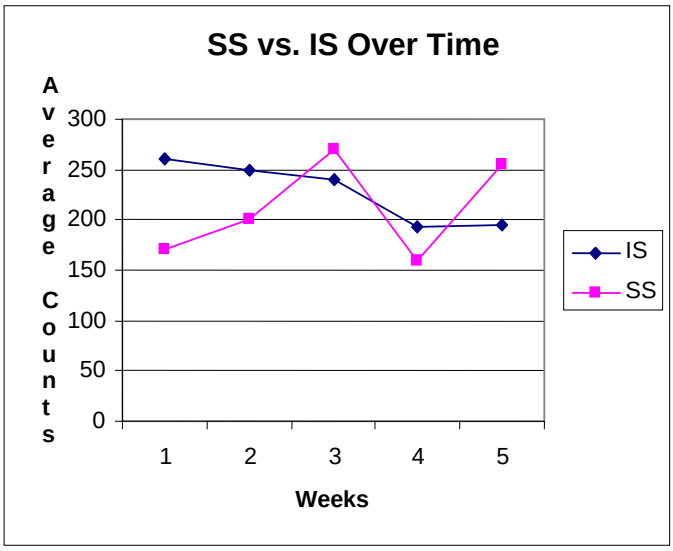

Fig. 1: Showing the comparism of activity between the sandwich and interceed side

Table 1: Showing comparative average count and activity ratio (SS = Sandwich Side; IS = Interceed Side)

	Pig No. 2		Pig No. 3		Pig No. 4		Pig No. 5		Pig No. 6	
Mandibles	L	R	L	R	L	R	L	R	L	R
Time (Weeks)	8 Weeks		11Weeks		13 Weeeks		17 Weeks		24 Weeks	
Component of Sandwich Unit	IS	SS	IS	SS	IS	SS	IS	SS	IS	SS
Size of Pixel										
Average Count	260	170	250	200	240	270	194	160	195	255
Activity Ratio	1:25		0:9		1:53		1:18		0:77	