Conceptual Mathematical basis for GIS Visualization Of Damage in Composite Laminates

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Introduction:

Composite laminated materials are formed by the combination of two or more materials in such a way that the composite materials are still distinguishable, and not fully blended. Composite materials take advantage of the different strengths and abilities of different materials (see Figure 1). Benefits of composites include reduced weight, improved fatigue resistance, reduced cost, and ability to make complex shapes. However, limitations include susceptibility to impact damage and difficulties to simulate and model theses damages.

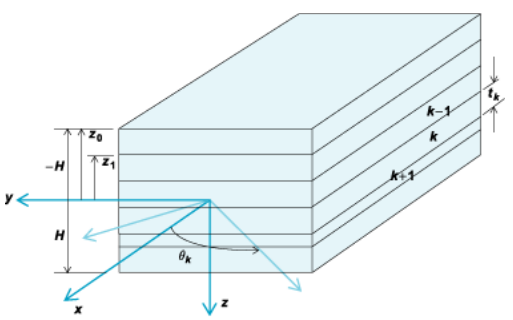

Figure 1

Techniques in Geographic Information Science, GIS, facilitate ability to store, manipulate and visualize large amount of spatially distributed data [3, 5]. GIS also provides special analysis tools that combine information characterized by location. However, the accuracy of the GIS results is not based on the quality of the graphical output. The errors we encounter in the analysis of cracks in laminated systems are mainly due to difference between a measured value or the "true" value and the estimated value with respect to a given observation in coordinate values of points and lines. It is understood, however, that the worth of a computed solution must be carefully weighed against these errors [1, 4]. Hence, in this concept paper we introduce mathematical concepts behind the analysis and visualization of cracks in laminated systems using GIS techniques. We also provide a rigorous mathematical proof to manage and evaluate the accuracy of these techniques.

Raster Representation:

In this paper, we assume that there are N_L numbers of rectangular laminated layers, and each layer is divided into N and M partitions respectively. The fact that a layer is a collection of grid points that cover the plane in a regular grid means that the each point can be considered as ordered in row and columns.

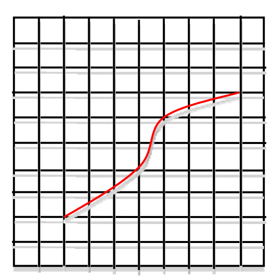

Figure 2

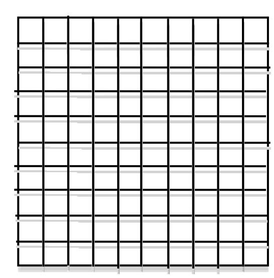

Figure 3

We adapted the definition of the raster geometry in GIS to the need of the small-scale representation of laminated layers and cracks on those layers. To represent a crack line, we need some form of spatial consideration. In general, the position of a crack line will fall somewhere inside one of the cells and crack lines intersect a series of cells, as shown in Figure 2. If a crack line intersects a cell at other locations than the nodes describing the cell, then the crack is represented by the nearest node defining one of the corners of the cell in which the line is located [2]. The representation is then according to Figure 3. The consequence of this representation is that the location of the crack will be shifted. However, we can prove that the average value of this error decreases as the partition size of the raster decreases. In other words, if *u(x(t),y(t))* and $u_h(x(t),y(t))$, are parametric functions representing original and approximated cracks respectively, we can show that

$$\int_C |u - u_h| \le Kh.$$

Where *h* is diagonal length of a rectangular cell, *C* is a curve of a crack and K is an arbitrary constant which depends on the function *u(x(t),y(t))*.

Matrix Representation:

Matrices are mathematical forms of shorthand. We can use matrices with techniques in GIS to manipulate and display cracks in new and explicit ways. Figure 4 demonstrates a composite material with five laminated layers that contains two cracks on the second and fourth laminates. At the bottom layer we can see both cracks on the same plane.

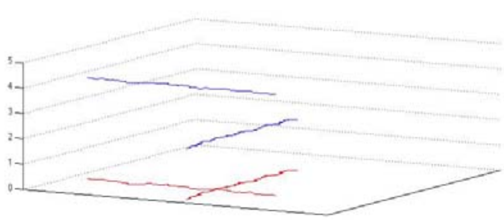

Figure 4

If a laminated plane is partitioned into *N* and *M* number of segments in *X* and *Y* directions, it creates a plane with *NM* number of cells and *(N+1)(M+1)* number of nodes. To analyze the cracks that appear in theses layers, we therefore use *(N+1)-by-(M+1)* matrix to represent each plane. If there are no additional geometric elements or cracks that appear on the plane, then all the elements of the matrix take a constant value (0 for example). Nodal values of the cracks are given by the approximation function $u_h(x,y,n)$, where $n = 1 \ldots N_L$ indicates location of the crack from the bottom.

As simulated in the Figure 4, cracks in the second and the fourth layers are given by the randomly generated functions $u_h(x,y,2)$ and $v_h(x,y,4)$. Two cracks at the bottom of the layer or the projected cracks to two dimensional plane is given by $u_h(x,y,0)+v_h(x,y,0)$. By interchanging *x* and *n* or *y* and *n*, of above function u_h , we can easily simulate vertical cracks. Error analyses for both horizontal and vertical cracks are identical.

One of the main advantages of this technique is the ability to create an interactive map of the composite material. Using the above matrix, we can scan the structure in any direction, zoom in or out, and change the nature of the information. Also, we can choose whether to see the individual plates, multiple plates and how they are affected by the cracks. Moreover, these mathematical concepts together with GIS techniques can be used to analyze variations of temperature, pressure, and other physical information on laminated composites.

References:

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