A NUMERICAL HOMOGENIZATION APPROACH FOR ELASTIC AND PIEZOELECTRIC COMPOSITES WITH RHOMBIC FIBER ARRANGEMENTS

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Introduction

Fiber reinforced composites get an increasing attention in the development of new materials. By controlling the manufacturing process, it is possible to get the desired material properties. Furthermore by inclusion of piezoelectric fibers in a resin matrix excellent composites can be designed for use in smart structures. With the recent advances in numerical modeling of composites, it is possible to predict the effective material properties of the composites.
A number of numerical and analytical methods have been developed to estimate the effective coefficients using homogenization methods. By micro-mechanical models based on unit cells the problem can be reduced on investigation of a periodic part of an infinite structure. But existing approaches are often restricted to certain types of arrangements. Mostly typical simple arrangements like square or hexagonal pattern have been investigated which result in an overall transverse isotropic behavior of the composite. An interesting goal is to create composites with orthotropic behavior in the transverse plane which can be achieved by rhombic fiber arrangements in connection with high volume fraction for the fibers. But nearly no results are published in literature for such patterns of fibers. Jiang [1] calculated with analytical methods effective shear coefficients for selected rhombic angles.
At our institute a general numerical homogenization technique for calculating effective material properties of composites with various fiber distributions has been developed [2]. Special procedures were used to create a comprehensive, highly automatic homogenization tool which combines pre-processing steps for geometrical modeling and applying of boundary conditions with finite element solution process. This paper is focused on special considerations for models with rhombic fiber pattern for elastic and piezoelectric composites.

Algorithm and Models

The numerical algorithm is based on a micro-mechanical unit cell model which contains the real distribution of inclusions. The unit cells represent a periodic array of the global structure. To ensure periodicity also after deformation appropriate periodic boundary conditions must be applied.
The basic idea for calculating effective material properties is that the strain energy stored in the heterogeneous system must be approximately the same like in the homogeneous system. With FEM for elastic case the averaged element strains $\bar{S}_{ij}$ and stresses $\bar{T}_{ij}$ are calculated and summed over all elements *k* of the unit cell.

$$\bar{S}_{ij} = \frac{1}{V}\sum_k S_{ij} V_k \; ; \quad \bar{T}_{ij} = \frac{1}{V}\sum_k T_{ij} V_k$$

Then from the following constitutive equations for such orthotropic case

$$\begin{bmatrix} \bar{T}_{11} \\ \bar{T}_{22} \\ \bar{T}_{33} \\ \bar{T}_{23} \\ \bar{T}_{31} \\ \bar{T}_{12} \end{bmatrix} = \begin{bmatrix} C_{11}^{eff} & C_{12}^{eff} & C_{13}^{eff} & 0 & 0 & C_{16}^{eff} \\ C_{12}^{eff} & C_{22}^{eff} & C_{23}^{eff} & 0 & 0 & C_{26}^{eff} \\ C_{13}^{eff} & C_{23}^{eff} & C_{33}^{eff} & 0 & 0 & C_{36}^{eff} \\ 0 & 0 & 0 & C_{44}^{eff} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55}^{eff} & 0 \\ C_{16}^{eff} & C_{26}^{eff} & C_{36}^{eff} & 0 & 0 & C_{66}^{eff} \end{bmatrix} \begin{bmatrix} \bar{S}_{11} \\ \bar{S}_{22} \\ \bar{S}_{33} \\ \bar{S}_{23} \\ \bar{S}_{31} \\ \bar{S}_{12} \end{bmatrix}$$

the effective elastic constants can be calculated by constructing different load cases in this sense that only one particular strain component is non-zero and all others are zero. This can be achieved by applying appropriate boundary conditions which produce pure tension and pure shear. E.g. for the calculation of C_{11}^{eff} only $\bar{S}_{11}$ may be non-zero. Then C_{11}^{eff} can be calculated from first row of constitutive equations by the ratio of $\bar{T}_{11} / \bar{S}_{11}$ and C_{12}^{eff} from the second row by the ratio of $\bar{T}_{22} / \bar{S}_{11}$ and analogous C_{13}^{eff} etc.
With coupled field equations for piezoelectricity in the matrix form of

$$T = C\,S - e\,E$$
$$D = e^T S + \varepsilon\,E$$

where *e* – piezoelectric constants, *D* – electrical displacements, *E* – electric field and *e* – dielectrical

constants, the above constitutive equations are extended by 3 rows and 3 columns.
All calculations are made with FE package ANSYS which provides with the included ANSYS Parametric Design Language (APDL) a convenient open interface for user specified input scripts.
The problem for the rhombic fiber pattern is that the simply created unit cell has not rectangular edges. In this case problems arise in applying the periodic boundary conditions. In our approach we extract a rectangular unit cell like shown in Fig. 1. It should be mentioned that the unit cells used for FE calculations are 3D (extended in the third direction) to calculate all coefficients for the three dimensional case. First the effective coefficients are calculated related to local coordinate system x_1'-x_2'. Finally by matrix rotation we get the effective coefficients for the global system.

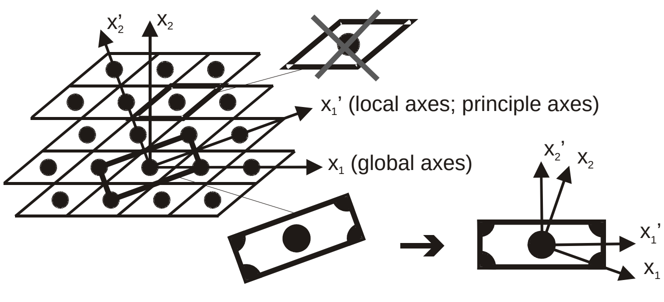

Fig. 1 Rhombic fiber arrangement

Results

For general verification of the algorithm the results for elastic case where compared with Jiang [1] who presented values for rhombic angles of 45 and 75 degrees. A very good agreement was found.
For a piezoelectric composite all coefficients where calculated for a rhombic angle range from 30 to 90 degrees and for various volume fractions. Fig. 2 shows selected effective coefficients in the global coordinate system. It can clearly be seen that for low rhombic angles a typical orthotropic behavior in the transverse plane is obviously. Furthermore the special cases for 60 degrees (hexagonal arrangement) and 90 degrees (square arrangement) show the typical transverse isotropic behavior.

Conclusions

A comprehensive tool for calculating effective material constants is introduced. Especially it is applied to elastic and piezoelectric composites with included fibers by rhombic arrangement. With this approach the excellent orthotropic behavior in transverse plane of such composites can be exhibited.

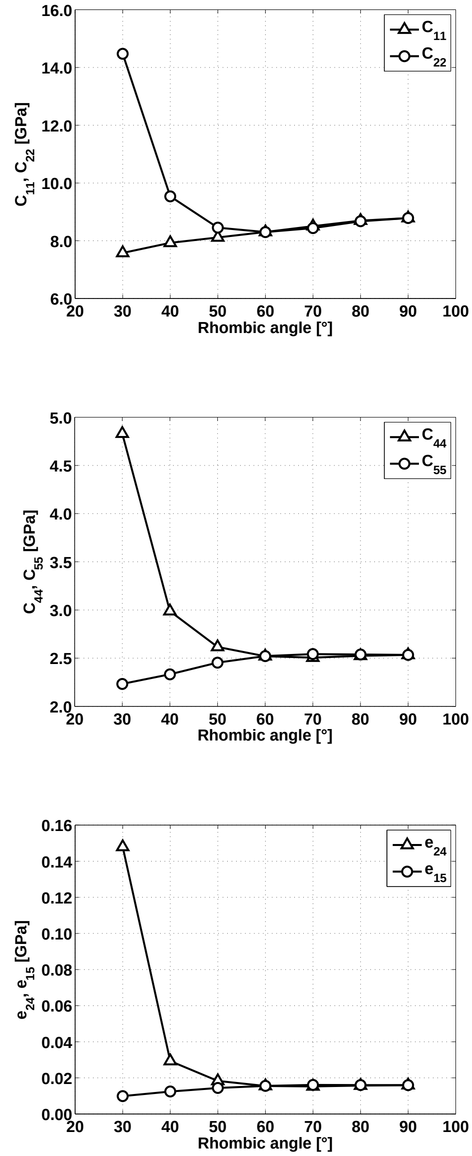

Fig. 2 Orthotropic behavior in transverse plane plotted for appropriate elastic and piezoelectric coefficients vs. change of rhombic angle with fixed fiber volume fraction of 0.4

References

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2. Berger, H., Kari, S., Gabbert, U., Rodriguez-Ramos, R., Bravo-Castillero, J., Guinovart-Diaz, R. A comprehensive numerical homogenisation technique for calculating effective coefficients of piezoelectric fibre composites. *Mat. Sci. and Eng. A*, **412** (2005) 53-60.