DYNAMIC ANALYSIS OF AN ORTHOTROPIC SHALLOW SPHERICAL SHELL ON PASTERNAK FOUNDATION

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Abstract
Larger amplitude free vibration of an orthotropic shallow spherical shell on Pasternak foundation has been studied by Paliwal et al. [1]. The authors in the present work, have extended the above work to analyse the effect of dynamic forces using theories of strain energy and simple harmonic motion. The period of vibration has been formulated in terms of geometrical parameters of the shell, foundation parameters, loads, non-dimensional deflection and non-dimensional amplitude characteristics. Numerical results are obtained for movable as well as immovable clamped edges. The effects of shell geometry and foundation parameters on frequency-amplitude characteristics are studied and plotted.

1. Introduction

Shells as structural elements result in reduced weight and economical use of materials. Shallow spherical shells are used in shell foundations, domes, process equipment and storage vessels. When they are used as underwater or buried structures, they interact with elastic media in their vicinity. Hence they are supported on elastic foundation. Orthotropic materials (composites) provide versatility and lightweight-ness in them. Such shells may be subjected to seismic forces. The seismic forces induce a sudden shift of the foundation of such shells. The inertia of the shells restrain them from moving, simultaneously with the foundation. This results in an elastic deflection of the shells that initiates a harmonic vibration in them. Hence, study of orthotropic shallow spherical shells resting on elastic media is of practical relevance. In this paper, the elastic medium is being represented by a two-parameter Pasternak foundation model.

2. Problem Formulation

A clamped shallow spherical shell on Pasternak foundation is shown in Fig. 1 in which the distance of middle surface of the shell from the plane of the base is given by

$$z = \frac{R^2}{2R_o}\left(1 - \frac{r^2}{R^2}\right) \qquad (1)$$

where z is the distance of middle surface of the shell from the plane containing edge, r is a cylindrical coordinate, R and R_o are the radius of the base circle and curvature of the shallow spherical shell, respectively.

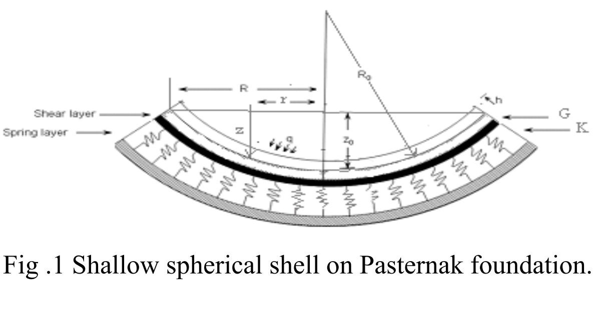

Fig .1 Shallow spherical shell on Pasternak foundation.

The response equation of the Pasternak foundation, in which the incompressible vertical elements deform by transverse shear only, is given by

$$p(r) = Kw(r) - G\nabla^2 w(r) \qquad (2)$$

where K and G are foundation modulus and shear modulus respectively, and w is the displacement in z-direction.

3. Analysis

The total potential energy of the system U is given by [2] as

$$U = \frac{D_1}{2}\iint\Bigg[(\nabla^2 w)^2 - \frac{2(\overline{K}-\nu_2)}{r}\frac{dw}{dr}\frac{d^2w}{dr^2} + \frac{12}{h^2}\left\{\overline{e}_1^{\,2} + \frac{1}{2}\beta\left(\frac{dw}{dr}\right)^2 + \frac{dw}{dr}\frac{dz}{dr}\right\}^2 - \frac{2qw}{D_1} + \frac{Kw^2}{D_1} + \frac{G}{D_1}\left(\frac{dw}{dr}\right)^2\Bigg]\, r\,dr\,d\theta \qquad (3)$$

in which $\overline{e}_1 = \dfrac{du}{dr} + \dfrac{\overline{\nu}u}{r} + \dfrac{1}{2}\left(\dfrac{dw}{dr}\right)^2 + \dfrac{dw}{dr}\dfrac{dz}{dr}$

and $\overline{K} = \sqrt{\dfrac{\nu_2}{\nu_1}} = \sqrt{\dfrac{E_2}{E_1}} = \sqrt{\dfrac{D_2}{D_1}}$

where ν_1 and ν_2 are Poisson's ratios, E_1 and E_2 are Young's moduli, D_1 and D_2 are flexural rigidities in orthogonal directions, h is shell thickness, β is a constant whose value for minimum potential energy is $\beta = 2\,\nu_1\nu_2$ and $\overline{\nu} = \sqrt{\nu_1\nu_2}$.

The clamped edge boundary condition is satisfied by assuming deflection w as

$$w = w_0\left(1 - \frac{r^2}{R^2}\right)^2 \qquad (4)$$

in which w_o is central deflection. Using the principle of minimum potential energy and satisfying the Euler's equation of calculus of variations, $\bar{e}_1$ is obtained as [2]

$$\overline{e}_1 = \left[\frac{16w_0}{R_0}\frac{\overline{\nu}R^{(1-\overline{\nu})}}{(\overline{\nu}+3)(\overline{\nu}+5)} + \frac{128w_0^2\overline{\nu}R^{(-\overline{\nu}-1)}}{(\overline{\nu}+3)(\overline{\nu}+5)(\overline{\nu}+7)}\right] \qquad (5)$$

Due to Eqs. (4) and (5), Eq. (3) yields

$$U = \pi D_1\Bigg[\frac{32w_0^2}{3R^2} + \frac{12R^2}{\overline{\nu}h^2}\left\{\frac{16w_0\overline{\nu}R^{(1-\overline{\nu})}}{R_0(\overline{\nu}+3)(\overline{\nu}+5)} + \frac{128w_0^2\overline{\nu}R^{(-\overline{\nu}-1)}}{(\overline{\nu}+3)(\overline{\nu}+5)(\overline{\nu}+7)}\right\} + \frac{16\beta R^2w_0^2}{5h^2R_0^2} - \frac{32\beta w_0^2}{5R_0h^2} + \frac{KR^2w_0^2}{10D_1} + \frac{8\beta w_0^4}{35R^2h^2} - \frac{qR^2w_0}{3D_1} - \frac{2Gw_0^2}{3D_1}\Bigg] \qquad (6)$$

4. Determining the Kinetic Energy

The kinetic energy (K.E.) of the shell on neglecting inertial effect due to radial displacement, is expressed by

$$K.E. = \frac{\rho h}{2} \iint \left(\frac{\partial w}{\partial t} \right)^2 r dr d\theta \qquad (7)$$

where ρ is density of the shell material. The clamped boundary conditions are satisfied by assuming transverse displacement as

$$w = A w_0 (\cos p_r t + \sin p_\theta t) \left(1 - \frac{r^2}{R^2} \right)^2 \qquad (8)$$

in which A is non-dimensional amplitude(≤1), p_p and p_θ have significance of angular velocity, and the terms $p_r t$, $p_\theta t$ are the angular displacements in radians at any time t. Functions $\cos p_r t$ and $\sin p_\theta t$ are the periodic functions that repeat, when the angular displacements reach 2π. The maximum kinetic energy exists at zero deflection, therefore $w = 0$ when $p_r t = \pi/2$ and $p_\theta t = \pi/2|_{n=0,1,2,....\infty}$. On substituting these conditions in the expression of $(\partial w/\partial t)$ and on rearranging, Eq. (7) yields

$$K.E. = A^2 \pi \rho h w_o^2 \frac{R^2}{10} (p_\theta - p_r)^2 \qquad (9)$$

On equating Eqns. (6) and (9), we get

$$p_\theta - p_r = 10 D_1 \left[\frac{32}{3} \left(\frac{w_0}{h} \right)^2 \left(\frac{h}{R} \right)^3 + \frac{192}{(\bar{\nu}+3)(\bar{\nu}+5)} \left(\frac{w_0}{h} \right) \left(\frac{1}{h} \right) \left(\frac{R}{R_o} \right) + \right.$$

$$+ \frac{1536 R^{-1}}{(\bar{\nu}+3)(\bar{\nu}+5)(\bar{\nu}+7)} \left(\frac{w_0}{h} \right)^2 + \frac{16\beta}{5} \left(\frac{w_0}{h} \right)^2 \left(\frac{R}{R_0} \right)^2 -$$

$$- \frac{32\beta}{5} \left(\frac{w_0}{h} \right)^2 \frac{w_0}{R_o} + \frac{8\beta}{35} \left(\frac{w_0}{h} \right)^2 \left(\frac{w_0}{R} \right)^2 + \frac{K w_o^2 R^2}{10 D_1}$$

$$\left. - \frac{q w_o R^2}{3 D_1} - \frac{2 G w_o^2}{3 D_1} \right] / \left[A^2 \rho h w_0^2 R^2 \right] \qquad (10)$$

For ($p_\theta \neq p_r$),

$$10 D_1 \left[\frac{32}{3} \left(\frac{w_0}{h} \right)^2 \left(\frac{h}{R} \right)^2 + \frac{192}{(\bar{\nu}+3)(\bar{\nu}+5)} \left(\frac{w_0}{h} \right) \left(\frac{1}{h} \right) \left(\frac{R}{R_o} \right) + \frac{16\beta}{5} \left(\frac{w_0}{h} \right)^2 \left(\frac{R}{R_0} \right)^2 + \right.$$

$$\left. + \frac{1536 R^{-1}}{(\bar{\nu}+3)(\bar{\nu}+5)(\bar{\nu}+7)} \left(\frac{w_0}{h} \right)^2 - \frac{32\beta}{5} \left(\frac{w_0}{h} \right)^2 \frac{w_0}{R_o} + \frac{8\beta}{35} \left(\frac{w_0}{h} \right)^2 \left(\frac{w_0}{R} \right)^2 \right]$$

$$= \frac{10 q w_o R^2}{3 D_1} + \frac{20 G w_o^2}{3 D_1} - \frac{K w_o^2 R^2}{10 D_1} \qquad (11)$$

The period of vibration T is equal to

$$T = \frac{(p_\theta - p_r)}{2\pi} \qquad \text{for } (p_\theta \neq p_r) \qquad (12)$$

5. Numerical Results

Numerical results are obtained for the shallow shells made of composite materials having the following values of various parameters.

For Fiber Reinforced Plastics (FRP): $\nu_1 = 0.0891$,

$\nu_2 = 0.07$ and $\bar{\nu} = \sqrt{\nu_1 \nu_2} = 0.079$

$\rho = 1600$ kg/m^3 $E_1 = 70$ GPa G= 50 GPa

$\lambda = KR^4/D_1 = 50, 100, 200$

$w_o/h = 0.5, 1.0, 2.0, 2.5$

h = 100 mm q = 10 kPa $R_o = 5$ m

R= 1m $\eta = R^2/R_o h$ $\mu = GR^2/D_1$

6. Discussions and Results

The plots of time period versus amplitude as a function of non-dimensional *wo/h* and foundation modulus λ are shown in Fig. 2 and 3 for clamped shell made of FRP.

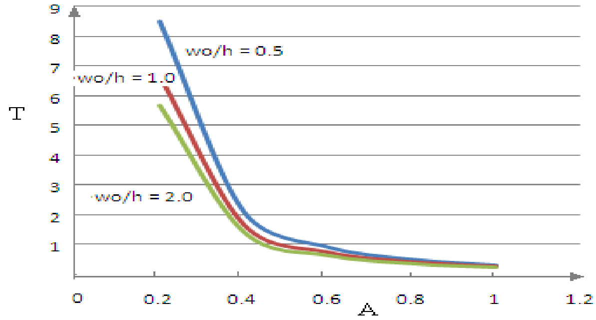

Fig. 2 Relation between time period T (s×10$^{4)}$ and amplitude A for clamped shell of FRP

These figures show that the time period of vibration reduces considerably near a critical value of amplitude A≈0.4. The value of T decreases for increasing value of *wo/h,* but decreasing value of λ.

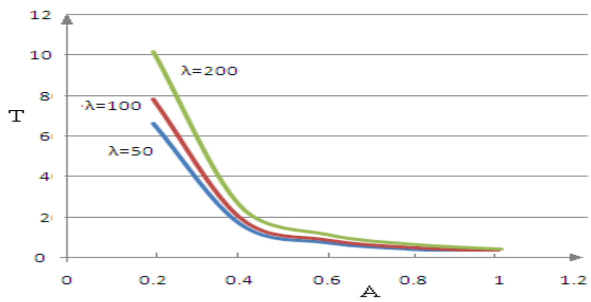

Fig. 3 Relation between time period T (s×10$^{4)}$ and amplitude A for clamped shell of FRP

7. References

(1) Paliwal D. N., Kanagasabapathy H., and K. M. Gupta, 'Vibrations of an orthotropic shallow spherical shell on a Pasternak foundation', Composite Structures 33(1995) 135-142.

(2) Paliwal D. N., Kanagasabapathy H., and K. M. Gupta, 'Large deflection of an orthotropic shallow spherical shell on a Pasternak foundation', International Journal of Pressure Vessel and Piping 62(1995) 117-122

(3) Bronell L.E. and Young E.H., 'Process Equipment Design', Wiley Eastern Ltd. (1997) p167.