Stress and Temperature Distribution in an Auxetic versus Non Auxetic Plates

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Introduction

Unlike most conventional materials auxetic materials expand perpendicular to an axis about which they are stretched. This gives such materials a variety of useful properties[1]. Under mechanical deformation, all auxetic materials have a common deformation characteristic to an ellipsoidal shape, referred to as the synclastic curvature. Consequently, a plate made of auxetic material will take up a dome-shaped configuration when bent, with no crimps induced by in-plane buckling. A normal material responds to this by attempting to shrink in the perpendicular direction, so the edges tend to curl upwards, producing a saddle-shaped surface. But in auxetic materials the response is to cause the edges to curve downwards, the same direction as the bending force. This double curvature characteristic of auxetic materials can be used to design and build domes of other structures with complex curvature and shapes, to moulding and shaping panels for car body parts or aircraft components, such as nose cones and these panels are often composites. To form a conventional composite panel into a dome-shaped double curve requires either machining or brute force, which can cause damage. Auxetic materials will bend in two directions at once without weakening the material bends over a double-convex surface, such as a dome, without any bonds breaking.

Mathematical Modeling

The purpose of the present research is to analyse development of temperature and thermal stress fields of a multilayered composite plate with alternative layers of material with negative versus positive Poisson's ratio. To better understand the influence of auxetic properties it is proposed here to calculate stress and temperature for auxetic and non auxetic single layer separately and compare the results. The constitutive equations for a homogeneous linear isotropic elastic solid in the absence of body forces and internal heat sources in the context of coupled thermoelasticity are given in [2]

$$\theta_{ii} = \frac{1}{\kappa}\frac{\partial \theta}{\partial t} - \frac{\rho}{k} h + \frac{E\alpha T_0}{(1-2\nu)k}\frac{\partial e_{ii}}{\partial t} \tag{1}$$

$$u_{i,jj} + \frac{1}{1-2\nu} u_{j,ji} + \frac{\rho}{\mu} b_i - \frac{2(1+\nu)\,\alpha}{(1-2\nu)k}\theta_{,i} = \frac{\partial^2 u_i}{\partial t^2} \tag{2}$$

$$\tau_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} + \beta\theta\delta_{ij} \tag{3}$$

The derivative of e_{ii} in equation (1) is coupling existing between the temperature and deformation fields and $\theta_{,i}$ is the deformation gradient. Assuming that both the displacement and the temperature fields are dependent on the field variable x and time *t* only, the displacement field has component only in *x*-direction, where x is measure across the plate and y and z are two remaining co-ordinates of the Cartesian rectangular system.

Initially at time, $t = 0$, we have

$$\Theta_j = 0\,,\ u_j = 0,\ \frac{\partial u_j}{\partial t} = 0,\ j = 0,1 \tag{4}$$

At the left surface $x = x_0$ of the plate, the heat flows from the atmosphere into the plate and the surface is stress free, we have boundary conditions as the following:

$$K\frac{\partial \Theta_0}{\partial x} + h_A(\Theta_A - \Theta_0) = 0,\ \tau_0 = 0 \tag{5}$$

At the right surface $x = x_1$ the appropriate boundary conditions are

$$K\frac{\partial \Theta_1}{\partial x} + h_B(\Theta_1 - \Theta_B) = 0\ ,\ \tau_1 = 0 \tag{6}$$

where, $\Theta_A = T_A - T_0$, $\Theta_B = T_B - T_0$, T_A, T_B are the absolute temperatures of the atmosphere on the left and right of the plate respectively, h_A, h_B are coefficients of heat transfer from the outer surfaces of the plate into the surrounding atmosphere and *K* is the thermal conductivity. The object of our interest is a plate subjected to a sudden uniform temperature applied at the face $x = x_0$ with temperature *T* and time evolution of both stress and temperature across the plate thickness.

Numerical Results

Using l^2/k_1 , l, T_0 and C_1 for the units of time, displacement, temperature and stress, we define these non-dimensional quantities as following:

$$x' = \frac{x_j}{l} \;, u'_j = \frac{u_j}{l}, t' = \frac{k_1}{l^2 \rho_1 C_1} t, \; \Theta'_j = \frac{\Theta_j}{T_0}, \tau' = \frac{\tau_j}{\lambda_1 + 2\mu_1} \qquad (7)$$

We use the Laplace transform and it's inverse to solve the simultaneous equations obtained upon substituting boundary conditions given by equations(3)-(6) in equations(1)-(3). Numerical results are obtained for the variation of temperature taking constant and initial temperature $T_A = 900K, T_B = 600K, T_0 = 300K$ and Poisson's ratio $\nu = 0.33$ for non-auxetic and $\nu = -.7$ in the case of auxetic material at non dimensional times, t. The variation in stresses and temperature with distance in a single layered auxetic and non-auxetic plate are shown in graphical form in Figures 1,2,3,4 and the values of relevant parameters for Steel and Copper Foam in SI units are given in Table 1.

Table 1		
Material Constant	Steel	Copper Foam
Young's modulus, E	200×10^9[Pa]	300×10^9[Pa]
Specific heat, C_e	0.560×10^3 J/kg ^{0}C	0.398×10^3 J/kg ^{0}C
Lame constant, μ	6.453×10^{10} N/m^2	7.330×10^{10} N/m^2
Lame constant, λ	9.5209×10^{10} N/m^2	-2.868×10^{11}N/m^2
Thermal exp.,α	17.7×10^{-6} /k^0	0.55×10^{-6} /k^0
Mass density, ρ	7.97×10^3 kg/m^3	8.93×10^2 kg/m^3
Thermal Cond., K	19.5 W/mk^0	70 W/mk^0
Poisson's ratio	0.33	-0.7

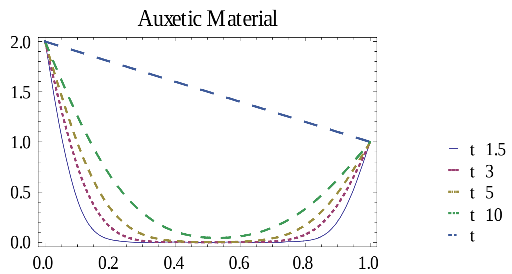

Figure 1, Temperature Variation

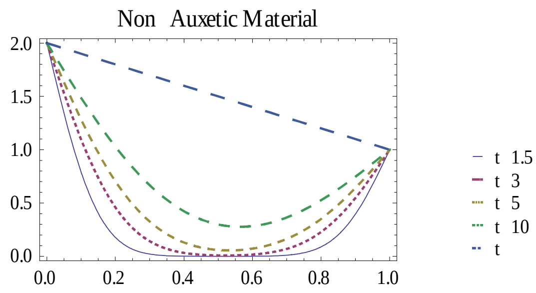

Figure 2, Temperature Variation

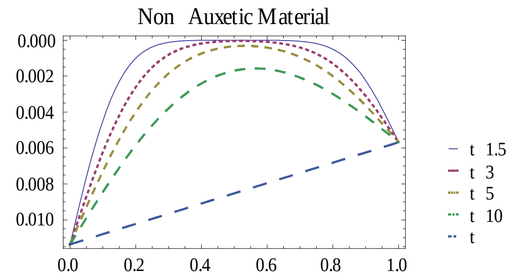

Figure 3, Stress component, τ_{yy}

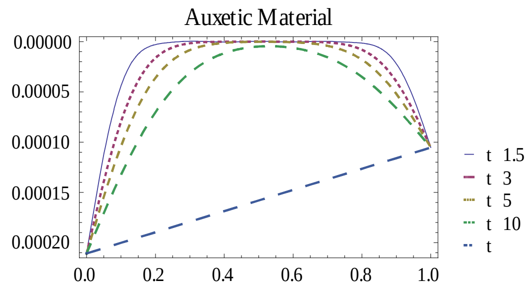

Figure 4, Stress component, τ_{yy}

Concluding Remarks

The results indicate that a further study of thermoelastic problems involving composite plates with non-auxetic/auxetic layers should be pursued.

References

1. Lakes, R.S. (1987a), Foam structures with a negative Poisson's ratio, Science, Vol. 235, pp. 1038-1040, 1987.
2. J.L. Nowinski, Theory of thermoelasticity and Applications.