COMPOSITE MECHANICS IN THE LAST FIVE DECADES

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Introduction

Modeling of physically three dimensional (3D) composite laminates were initially accomplished by using the classical two dimensional (2D) elastic plate and shell theories (CLT) based on Kirchhoff hypothesis. An individual layer is assumed to be homogeneous and orthotropic (and thus the material properties are assumed to remain constant in each layer) while the laminate is heterogeneous through the thickness and generally anisotropic. It was soon realized that CLT was unsuitable for laminates in which transverse shear deformation and transverse normal deformation energies were significant. The so-called first-order shear-deformation theories (FOST) of Reissner [1] and Mindlin [2] which are based on assumed stress and displacement fields respectively did include a constant transverse shear strain/stress through the laminate thickness. A shear correction coefficient, which is somewhat arbitrary, is used to correct the transverse shear strain energy of deformation. These limitations of CLT and FOST led to the development of higher-order shear-deformation theories (HOST) which could include the consideration of realistic parabolic variation of transverse shear stresses through the laminate thickness, distortion of the transverse normal, transverse normal deformation and the complete 3D material constitutive law [3-11].

Transverse Inter-laminar Stresses

Stress, free vibration and buckling analyses of laminated composites and sandwiches are carried out by various 2D theories. The in-plane stresses are first computed in the first phase and the transverse interlaminar stresses are then estimated by integrating 3D equilibrium equations of elasticity along the laminate thickness in the second post-processing phase and there are serious limitations in this procedure. The estimates are not only inaccurate but the methods are unreliable and the whole methodology lacks robustness. The root cause for this situation lies firstly in the mathematical model for the integration of the transverse shear stresses. The mathematical models are improperly posed boundary value problems (BVPs) and secondly the error propagation in the numerical evaluation of the high derivatives of displacements. Therefore, accurate and reliable evaluations of the transverse stresses have not been addressed adequately till very recently.

Significant contributions have been made in the development of direct numerical transient analysis procedure in the last three decades. Most of these procedures are based on semi-discretization methods in which the space dimension is discretised by the FE method. Such a strategy transforms the governing system of partial differential equations (PDEs) to a system of ordinary differential equations (ODEs) in time.

An attempt is made here to extend this methodology to elastostatic problems ranging from 3D solids to 2D plates and shells to one dimensional (1D) beams and arches and whose behavior is mathematically formulated as a two-point BVP governed by a set of liner first-order ODEs.

$$\frac{d}{dz}\mathbf{y}(z) = \mathbf{A}(z)\mathbf{y}(z) + \mathbf{p}(z) \tag{1}$$

in the domain z1< z < z2. y(z) is an n-dimensional vector of dependent variables, A(z) is an n x n coefficient matrix and p(z) is an n-dimensional vector of nonhomogeneous (loading) terms. Any n/2 elements of y(z) are prescribed at the two ends, z = z1 and z2 as boundary conditions. It is clearly seen that mixed and/or nonhomogeneous boundary conditions are easily admitted during solution.

BVPs in ODEs, not only describe 1D elastostatic problems exactly but also 2D and 3D problems approximately whose behavior is governed by a system of PDEs. The use of the well-known beam functions, Fourier series, harmonic analysis, etc. for dimensional reduction is a well-established procedure in mechanics, which helps in transforming PDE into ODE system. It is sometime convenient to have a discrete dependent vector y(z) to be function of only one of the independent coordinate z while carrying out the finite element discretization of the prismatic domain defined by independent coordinates x and y. The basic approach to the numerical integration of the BVP defined by Eq. (1) is to transform the given BVP into a set of initial value problems (IVPs) – one particular (nonhomogeneous) and n/2 complimentary (homogeneous). The solution of the original BVP defined by Eq. (1) is obtained by forming a linear combination of one nonhomogeneous and n/2 homogeneous solutions so as to satisfy the boundary conditions at z = z2. This gives rise to a system of n/2 linear algebraic equations, the solution of which determines the unknown n/2 components of the vector of initial values y(z1). Then a final numerical integration of Eq. (1) with completely known initial vector of dependent variables y(z1) produces the desired results.

Conceptualizing a finite element discretization in the lamina plane, a set of implicit first order ODEs is obtained. The solution vector of which consists of a set of fundamental dependent quantities whose number equals the order of the PDE system times the number of discrete finite element mesh nodes. Further, this fundamental set of quantities for any node consists of stress components and the corresponding displacements on the lamina plane. As noted earlier the availability of efficient, accurate and above all proven robust ODE numerical integrators for IVPs helps in obtaining the fundamental set of dependent quantities at all the nodal points through the thickness. Once the fundamental set is known, the auxiliary set of dependent quantities over the entire nodal set is computed simply by substitution of the values of the fundamental set of variables on the right hand side of algebraic expressions node-by-node. Initial experience with this novel formulation and the associated computational methodology is encouraging.

The proposed approach seems to have not got the attention that it deserves and we claim to be the first one to present a few non-trivial solutions. Few results obtained through present development are presented in Tables 1-3 [12-16]. The most significant advantage of this general methodology lies in the fact that both displacements and transverse stresses are evaluated simultaneously with same degree of accuracy through the numerical integration processes.

Conclusion

Various 2D simple theories for analysis of composites and sandwiches are discussed. Further, a novel partial discretization methodology is discussed in short. It is first of its kind of mixed model which is based on solution of two-point BVP governed by coupled first-order ODEs through thickness of laminates. Good agreement of the present results with the elasticity solution is observed. The most significant advantage of the present development lies in the fact that both displacements and transverse stresses are evaluated simultaneously at the finite element node with the same degree of accuracy through the numerical integration process and thus eliminating the post-processing module for calculation of transverse stresses from in-plane stresses.

Table 1 Normalized in-plane and transverse stresses and mid-plane displacement of simply supported 0°/90°/0° composite plates

s	Source	$\overline{\sigma_x}$	$\overline{\tau_{zx}}$	$\overline{w}$
4	Present	0.7556	0.2583	2.0046
	Elasticity	0.8010	0.2560	-
	HOST	0.7610	--	1.9260
10	Present	0.5750	0.3550	0.7471
	Elasticity	0.5900	0.3570	--
	HOST	0.5850	--	0.7176

Table 2 Normalized in-plane and transverse stresses and mid-plane displacement of simply supported -15°/15° composite plates

s	Source	$\overline{\sigma_x}$	$\overline{\tau_{zx}}$	$\overline{w}$
4	Present	0.6739	0.3652	1.7520
	Elasticity	0.6545	0.3145	1.7059
10	Present	0.5701	0.3849	0.8568
	Elasticity	0.5633	0.3421	0.8027

Table 3 Normalized in-plane and transverse stresses and mid-plane displacement of simply supported 0°/90° composite plates under cylindrical bending

s	Source	$\overline{\sigma_x}$	$\overline{\tau_{zx}}$	$\overline{w}$
4	Present	0.2325	0.6983	4.6826
	Elasticity	0.2397	0.6805	4.6953
10	Present	0.1952	0.7343	2.9503
	Elasticity	0.1983	0.7268	2.9538

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