COMPOSITE BOLTED JOINTS ANALYSIS BY MEANS OF THE COMPLEX POTENTIAL METHOD

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Introduction

The increasing use of composite structures in aircraft engineering raises the question of efficient joining technology. Besides bonding, bolted joints are one of the traditional connection methods. Due to the involved stress concentrations, bolted joints are potential weak locations and require corresponding attention in the design of the underlying structure.
The aim of the presented work is to analyze a joint-laminate specimen within the framework of the classical laminate theory [1] and the complex potential method by Lekhnitskii [2].

Method

Typically in reality a bolt joint test specimen has the form of a rectangle with a circular hole, as depicted in Fig. 1. In our method, the external boundary of the specimen is approximated by an ellipse Γ_{out} with major axis $2a$ equal to the length l of the specimen and minor axis $2b$ equal to its width w. This simplification is well justified since the corners of the specimen do not have any significant influence on the stress distribution on the notch border, which is critical with respect to potential failure. The diameter of the notch Γ_{in} is d and its centre is shifted by a-e in the direction of applied force with respect to the centre of the outer ellipse.

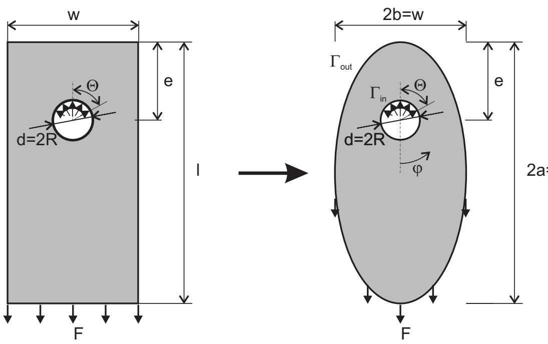

Fig. 1 The geometry of the specimen and its approximation

The action of the bolt on the laminate is approximated by a sinusoidal distribution of normal load over a given angle Θ. First, two complex variables ζ_{in} and ζ_{out} are introduced which map the boundary contours in the mapped plane to a unit circle. The complex potentials are represented by the series

$$\phi_j = c_j \ln \zeta_{\mathrm{in},j} + \sum_{n=1}^{\infty} c_{jn} \zeta_{\mathrm{in},j}^{-n} + \\ + \sum_{n=1}^{\infty} d_{jn} \left(\zeta_{\mathrm{out},j}^{n} + t_j^n \zeta_{\mathrm{out},j}^{-n} \right), \; t_j = konst.$$

The first part of this expansion corresponds to the solution of the problem of a plate with an elliptic opening loaded along its edge, the second part to the problem of an elliptical plate loaded along its edge [2]. A similar approach has already been successfully used in [3] for the analysis of an elliptically reinforced circular hole in an anisotropic plate or laminate and in [4] for the analysis of external patch repairs of laminate plates. To complete the solution we need to determine the coefficients c_j, c_{jn} and d_{jn} by taking into account the underlying boundary and continuity conditions. Substituting the series representation of the complex potentials into the underlying boundary conditions expressed in terms of the total force acting on a boundary arc, expanding all arising terms as a Fourier series of the angle φ introduced in Fig. 1, truncating all series after N terms and comparing the corresponding coefficients, we arrive at a well posed system of $4N$ complex linear equations for the coefficients c_j, c_{jn} and d_{jn}. From the now known complex potentials all other field variables can be determined according to [1] and [2].
So far, the contact angle Θ enters the analysis as a pre-assumed input quantity. In order to get a realistic contact angle an iterative analysis scheme has been implemented by which Θ can be identified in a few steps.

Results and comparison with the FEM

In order to asses the efficiency of the present method and to verify the assumption that the effect of the corners on the stress distribution in the neighborhood of the notch is negligible, a series of reference finite element computations has been conducted for the rectangular specimen (right part of Fig. 1). These computations were performed with the commercial FE-Programme ABAQUS using the four-node quadrilateral shell elements S4. First, convergence studies of both FEM and the present method have been carried out. The studies reveal that in the case of the FEM a relatively fine mesh in the neighborhood of the notch and a correspondingly high number of degrees of freedom (in the order of 100,000 in our model) are necessary in order to obtain the results with about 1% accuracy. On the contrary, for the present method, only 14 terms in the expansion of the complex potentials i.e. 112 degrees of freedom suffice for the same accuracy. The results of the convergence study on a typical specimen are shown in Fig. 2.

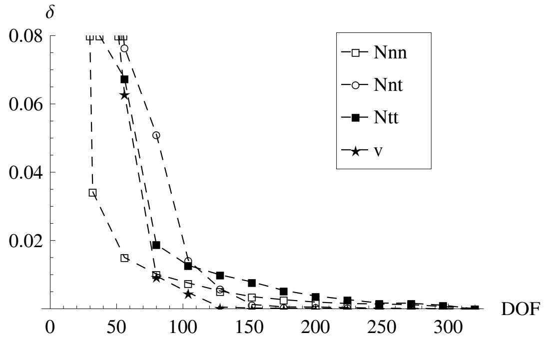

Fig. 2 Convergence of the present method

As an example of a comparison of the present method with FEM, Fig. 3—4 show the radial and axial displacement on the notch border as well as the tangential section force $N_{\varphi\varphi}$ normalized by the section force $N_0=F/w$ corresponding to an unnotched plate of width w loaded by the distributed force F. Obviously, the agreement between the corresponding results is very good. The results of the present approach, however, require much less computational effort in comparison to the finite element method.

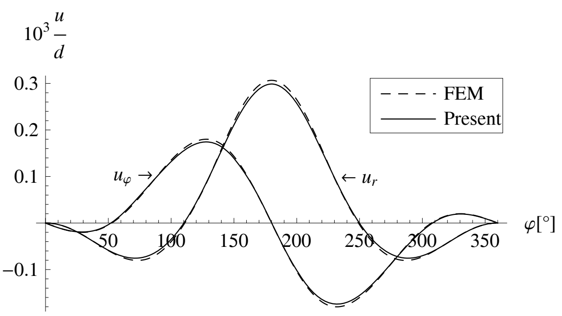

Fig. 3 Radial u_r and tangential u_φ displacement on notch border for $[\pm 45°]_s$, w/d=3

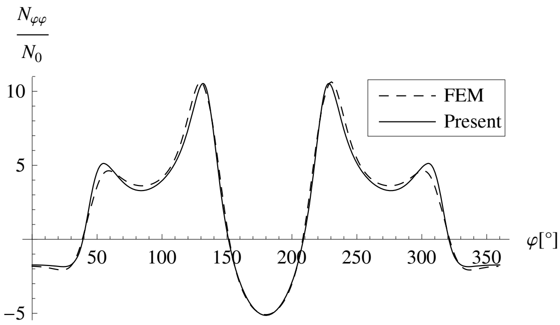

Fig. 4 Tangential section force $N_{\varphi\varphi}$ on notch border for $[\pm 45°]_s$, w/d=3

Finally, the influence of the edge distance on the theoretical strength according to the Tsai-Wu criterion has been investigated. Fig. 5 shows the maximal Tsai-Wu index over all plies plotted along the notch border. We observe that e.g. for *w/d=3* the maximal Tsai-Wu index is almost twice as high as in the case of an infinite plate which corresponds to half the critical force and with increasing *w/d* ratio the Tsai-Wu index converges to that of the infinite plate.

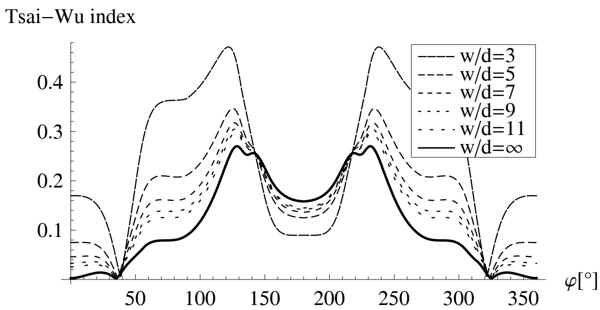

Fig. 5 Maximal Tsai-Wu index on the notch border for [±45°]s for various values of the w/d ratio including the infinite plate

Conclusion

To summarize, the presented method based on the Lekhnitskii complex potential theory yields stresses and displacements in the bolt-laminate specimen at low computational costs and high accuracy.

References

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