STRESS CONCENTRATION AROUND A CIRCULAR HOLE IN A FG PLATE UNDER BIAXIAL LOADING

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Introduction

Functionally graded materials (FGMs) has been received extensive scientific attentions recently [1]. These nonhomogeneous composites have enormous applications in science and technology, see e.g. [2]. Most of the analytical investigations on FGMs are on bounded geometries. For instance there has been a few contributions on finding stress concentration factor around a circular hole in an infinite plate made of FGMs.

Yang et al. [3] investigated the stress field around a circular hole in a FGM plate. They used piece-wise homogeneous layers and complex variable methods, the plate was decomposed into N rings with equal thickness and constant material properties. The stress concentration factor for different elastic variations was calculated.

In this paper, The stress concentration factor around a circular hole in an infinite plate made of a FGM subjected to biaxial loading is considered. Poisson's ratio is held constant and Young's modulus is considered to vary across the radius. It is noted that, power law functions are not suitable to define the spatial variation of Young's modulus in an infinite plate. Thus, an exponential function is used to define Young's modulus.

Formulation of the Problem

Fig. 1 shows an infinite plate with a circular hole of radius a. The plate is subjected to a biaxial stress σ_0. Polar coordinate system is used to calculate the stresses. The radial coordinate, r is normalized with respect to a and a dimensionless radius is then defined as $\rho = r/a$. The elastic solid is made of a functionally graded material so that Young's modulus E solely changes with radius. Power law functions are not suitable to define the spatial variation of Young's modulus in an infinite plate. Thus, an exponential function is used to define Young's modulus as below

$$E(\rho) = E_0 \exp(\eta \rho^s), \tag{1}$$

where η and s are parameters of nonhomogeneity and they are adjustable. For the sake of simplicity, Poisson's ratio ν is held constant. The problem is axisymmetric and the

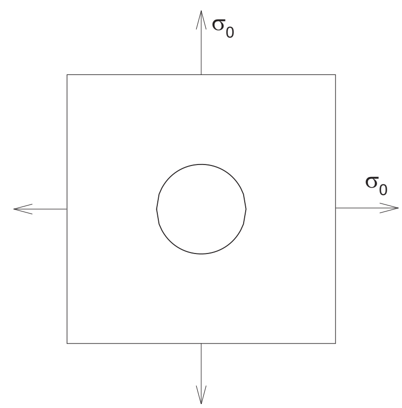

Figure 1: A circular hole in an infinite plate made of a FGM subjected to biaxial tension σ_0

compatability equation is written as [4]

$$\frac{\partial(\rho \epsilon_\theta)}{\partial \rho} = \epsilon_\rho. \tag{2}$$

The plate is in plane stress and Hooke's law is stated as below

$$\begin{aligned} \epsilon_\rho E(\rho) &= \sigma_\rho - \nu \sigma_\theta, \\ \epsilon_\theta E(\rho) &= \sigma_\theta - \nu \sigma_\rho, \end{aligned} \tag{3}$$

where $E(\rho)$ is defined in equation (1). Herein, stress function p is defined such that it satisfies the equilibrium condition [4] and they are written as

$$\sigma_\rho = \frac{p}{\rho}, \quad \sigma_\theta = \frac{dp}{d\rho} \tag{4}$$

Substituting the stress components (4) into the Hooke's law (3), and putting the resultant strains into the compatability equation (2), the governing differential equation for the stress function can be written as

$$\rho^2 \frac{d^2 p}{d\rho^2} + (1 - s\eta\rho^s)\rho \frac{dp}{d\rho} - (1 - s\nu\eta\rho^s)p = 0. \tag{5}$$

The periphery of the hole is stress free and the radial stress is equal to σ_0 at infinity; therefore, the boundary conditions for this problem are

$$p(1) = 0, \qquad \lim_{\rho \to \infty} \frac{p}{\rho} = \sigma_0. \tag{6}$$

The solution to the governing differential equation (5), satisfying the boundary conditions (6), is

$$\frac{p}{\sigma_0} = \left\{ \rho M(a_1, b_1, \eta\rho^s) - \frac{M(a_1, b_1, \eta) M(a_2, b_2, \eta\rho^s)}{\rho M(a_2, b_2, \eta)} \right\}, \tag{7}$$

where $M(a, b, x)$ is a Kummer's function and its properties can be found in mathematical references see e.g. [5] and the constants are defined as below

$$\begin{aligned} a_1 &= (1-\nu)/s, & a_2 &= (-1-\nu)/s, \\ b_1 &= (s+2)/s, & b_2 &= (s-2)/s. \end{aligned} \tag{8}$$

The stress concentration factor K is defined as the ratio of the maximum hoop stress which occurs at $\rho = 1$ to the reference stress σ_0 [6]. Using equations (4), (7) and some algebra, the factor K is calculated and written as

$$K = \frac{2\exp(\eta)}{M(a_2, b_2, \eta)}. \tag{9}$$

Considering homogeneous materials, i.e. $\eta = 0$, then $M(a_2, b_2, 0) \longrightarrow 1$ see e.g. [5] and the factor K reduces to two [6].

Results

Let us set $\nu = 0.35$. Fig.2 shows the stress concentration versus η for various values of s. Stress concentration is equal to two for all the graphs when $\eta = 0$ regardless of the parameter s. It is noted that for $\eta > 0$ the stress concentration factor is greater than two and vice versa for $\eta < 0$. This is because of the fact that when $\eta > 0$, Young's modulus around the hole is bigger than at infinity. This results to a bigger hoop stress around the hole; consequently, the stress concentration factor raises and it increases in comparison with the case of homogeneous materials. In the same way for the setting $\eta < 0$, since Young's modulus is smaller around the hole, so that the hoop stress is smaller and the parameter $K < 2$. It is obvious from the graphs that in the region $\eta > 0$, the stress concentration factor reaches higher values for bigger $|s|$ values. In addition, the bigger the $|s|$ value is, the smaller the Young's modulus is at infinity; thus, the material is stiffer around the hole and ultimately the stress concentration factor is higher and vice versa for the region $\eta < 0$.

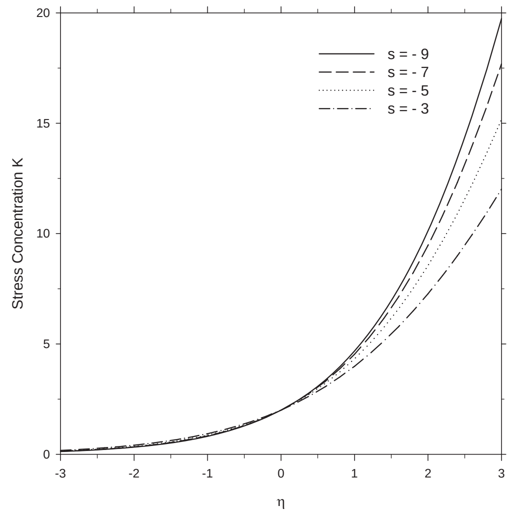

Figure 2: Stress concentration factor, around a circular hole in an infinite plate made of a functionally graded materials, versus the parameter η for different values of s

Conclusions

The stress concentration factor around a circular hole in an infinite plate made of a functionally graded material subjected to biaxial tension is investigated. The governing differential equation in terms of stress function is derived. The stress concentration factor is then calculated. It has been found that when the material used around the hole is stiffer than the material used distances far from the hole, the stress concentration factor is greater than the homogeneous case and vice versa.

References

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