SHAPE OPTIMIZATION OF FIBER CROSS SECTIONS IN UNIDIRECTIONAL COMPOSITES

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This paper presents a methodology to apply optimization techniques to the problem of tailoring the microstructure of a composite to achieve a desired or predetermined mechanical response. In particular, the cross sectional shapes of the reinforcing fibers of a polymeric matrix composite are modified to achieve a desired or target transverse viscoelastic response. The composite used is a carbon reinforced, polymeric matrix composite. The polymeric matrix consists of a commonly used epoxy resin, whose experimental creep compliance is reported in [1]. A theoretical model of this experimental creep compliance was developed and presented in [2]. This theoretical model uses a system consisting of an arbitrarily large number of Maxwell elements in parallel. A Maxwell element consists of one spring and one dashpot connected in series (see e.g. [3,4]). as shown in Figure 1b. The viscoelastic theory used in all of these references is that developed in [5]. The theoretical model used in [2] was calibrated to fit the experimental creep compliance of the epoxy resin using a state of the art sequential quadratic programming (SQP) technique [6]. For comparison purposes, a zero-order optimization technique was also used to calibrate this model in [5]. In this paper, we use the same SQP, and zero-order optimization techniques but apply them to the more difficult problem of shape optimization.

Shape Optimization Problem Description

The objective is to achieve the cross-sectional fiber shape(s) that exhibit a desired or predetermined mechanical response. The fiber shape is modeled using the following simple model:

$$\frac{x^\alpha}{a^\alpha} + \frac{y^\beta}{b^\beta} = 1$$

Shape variables can be *a*, *b*, α, and β. The composite is modeled using the Generalized Method of Cells (GMC) [7]. As a consequence, it is only necessary to model a representative volume element (RVE) or unit cell (see Figure 2). As in [2], the author's adaptation of a sequential quadratic programming (SQP) optimization technique was used. The objective function is taken as the L^2-norm of the difference between the numerically calculated response and the target one. SQP techniques require the use of derivative information that are obtained by means of finite differences. This derivative information is computationally expensive since it requires multiple objective function evaluations. Each function evaluation entails a complete simulation in time of the creep process. Still, the calibration problem studied in [2] and [4] is a relatively simple parameter optimization problem. In contrast, a shape optimization problem requires (in addition to a complete simulation in time), the re-meshing or re-gridding of the domain of definition of the problem (a unit cell in this case, see Figure 2). GMC was chosen in the present study since it is computationally more efficient than other micromechanical methods like the Strain Compatible Volume Averaging (SCVA) method or the finite element method.

Numerical Results

The result of the numerical tests are shown in Table 1 and Figures 1 and 2. The initial and target creep compliances are shown in Figure 3. Despite the fact that changes in fiber shape in the GMC unit cell model are rather discrete in nature (this is despite the 10,000 subcell resolution used in the present study--see Figures 1 and 2), the SQP technique converges to an optimal shape. The zero-order optimization technique, namely the *simplex* method of Nelder and Mead (also known as the polytope method [4, 6]), converged to a suboptimal point.

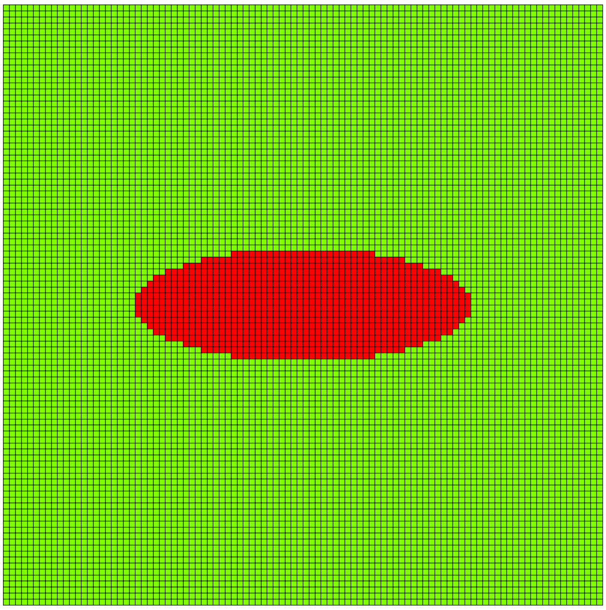

Figure 1. Unit cell with an elliptical fiber, used as initial fiber shape. 100 by 100 model = 10,000 subcells.

Table 1. Performance of the SQP and the polytope optimization methods.

Method	SQP	Polytope
Iterations	6	103
Function Evaluations	37	92
Derivative Evaluations	6	---
Reflections	---	103
Expansions	---	86
Contractions	---	71

Conclusions

It was found from the numerical tests that it is possible to use optimization techniques to find the optimal cross-sectional shape of the reinforcing fibers of a composite material to achieve a given or desired mechanical response. Despite the discrete nature of the discretization method used in this study (the generalized method of cells (GMC)), the SQP technique proved superior than the polytope method (a zero order technique). Further studies are required to develop methodologies that can accommodate more complex models of the fiber shapes than those used in the present study.

References

[1] Aboudi J., ``*Mechanics of Composite Materials – A Unified Micromechanical Approach*". Elsevier, Amsterdam, 1991, Chap. 4.

[2] Orozco, C.E. **"**Optimal Tailoring of a Viscoelastic Model for Polymeric Matrix Composites**"**. *Proceedings of the 12th International Conference on Composites/Nano Engineering, ICCE/12"*. Tenerife, Canary Islands, Spain, August 8-14, 2005.

[3] Orozco C. E. and Pindera M-J.,``Viscoelastic Analysis of Multi-Phase Composites Using the Generalized Method of Cells". *AIAA Journal*. Vol. 40, No. 8, August 2002, pp. 1619-1626.

[4] Orozco, C.E. **"**A Comparison of Optimization Tehcniques for a Modeling Problem in Viscoelasticiy**"**. *Proceedings of the 14th International Conference on Composites/Nano Engineering, ICCE/14"*. Boulder, Colorado, July 2-8, 2006.

[5] Saleeb A. F. and Arnold, S. M., ``A General Reversible Hereditary; Constitutive Model: Part I Theoretical Developments". *NASA TM 107493*, 1997.

[6] Gill P.E., Murray W., and Wright M. H., ``Practical Optimization", Academic Press, 1981, Chap. 4.

[7] Paley M. and Aboudi J., ``Micromechanical Analysis of Composites by the Generalized Method of Cells". *Mechanics of Materials*. 1992, Vol. 14. pp. 127-139.

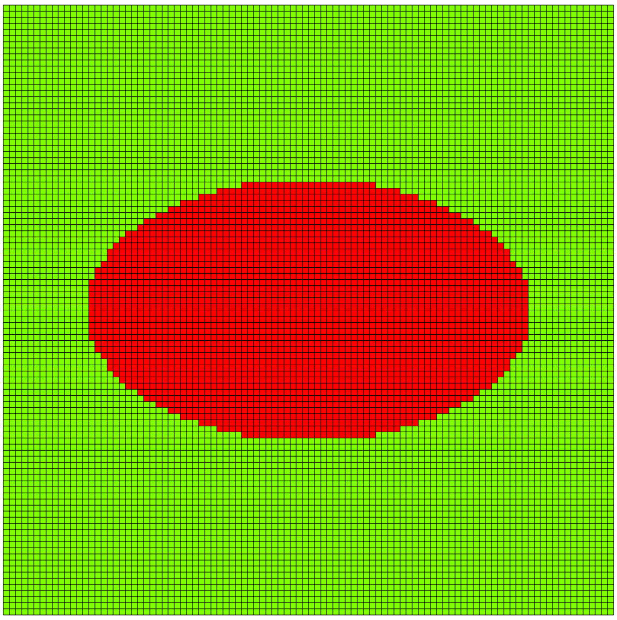

Figure 2. Unit cell with an elliptical fiber, used as target fiber shape. 100 by 100 model = 10,000 subcells.

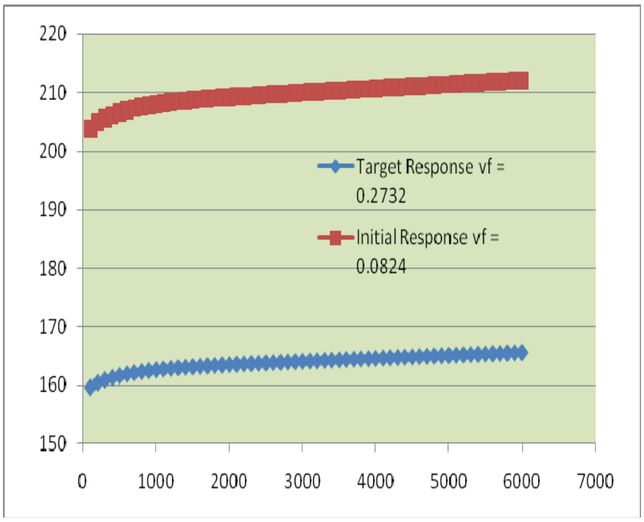

Figure 3. Target and Initial Creep Compliances (1/1.0E +06 MPa) versus time in seconds for the composite used for the numerical experiments.