SHAPE OPTIMIZATION OF TWO PHASE COMPOSITES WITH A HOLE

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Introduction

There is couple of approaches for shape optimization of composite structures with unidirectional fibers. They differ with the choice of cost functional, which assign the target intention of a potential user of the structure. It was proved in paper [1] that the requirement of minimum Lagrangian with constraint leads to minimum stresses and in the same time to minimum displacements. In the latter paper boundary element method was used as it does not depend on a domain meshing. In order to formulate a problem of shape optimization of general composite and to use boundary elements the trick published in [2] can be the way leading to reasonable integral formulation. In [3] this approach was used for homogenization of debonding composites with prescribed displacements. Here a model by Suquet, [4], serving for description of periodic composites enables us to formulate the necessary integral equations

Localization using BEM

The localization problem in 2D is posted as:

$$\sigma_{ij} = L_{ijkl}(\boldsymbol{y})\varepsilon_{kl}(\boldsymbol{y}) = L_{ijkl}(\boldsymbol{y})[E_{kl} + \hat{\varepsilon}_{kl}(\boldsymbol{y})]$$
$$\frac{\partial \sigma_{ij}}{\partial y_j} = 0, \qquad \text{periodic boundary conditions} \tag{1}$$

where E_{kl} are unit impulse strains, $\hat{\varepsilon}_{kl}$ are components of fluctuating strains, and (1) is to be taken in the sense of distribution, $i, j, k, l = 1,2$. For simplicity, symmetric unit cell is considered and therefore the boundary conditions for unit impulses obey the same situation as that described in [4]. It appears that on the interface Γ^C between phases tractions $p^C \equiv \{p_1^C, p_2^C\}$ appear depending on E_{11} as:

$$p_1^C = (L_{11}^{\mathrm{m}} - L_{11}^{\mathrm{f}})n_1 \ , \ p_2^C = (L_{12}^{\mathrm{m}} - L_{12}^{\mathrm{f}})n_2 \tag{2a}$$

on E_{22} as:

$$p_1^C = (L_{12}^{\mathrm{m}} - L_{12}^{\mathrm{f}})n_1 \ , \ p_2^C = (L_{22}^{m} - L_{22}^{\mathrm{f}})n_2 \tag{2b}$$

and on E_{12} as:

$$p_1^C = (G^{\mathrm{m}} - G^{\mathrm{f}})n_2 \ , \ p_2^C = (G^{\mathrm{m}} - G^{\mathrm{f}})n_1 \tag{2c}$$

and the unit overall strain impulses are introduced successively, i.e. one of their components is one and the other zero. If one phase is the hole in (2) quantities with f are put equal to zero.

Integral equation corresponding to (1) can be written as:

$$c_{mn}(\xi)\hat{u}_n(\xi) = -\int_{\partial\Omega} p_{mi}^*(\boldsymbol{y},\xi)\hat{u}_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) + \int_{\partial\Omega} u_{mi}^*(\boldsymbol{y},\xi)\hat{p}_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) + \int_{\Gamma^C} u_{mi}^*(\boldsymbol{y},\xi)p_i^C(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) \tag{3}$$

where the components with asterisk are known kernels, ξ is either inside of the matrix ($c_{mn} = \delta_{mn}$) or ξ belongs to the boundary, involving the interface Γ^C , $\hat{p}_i(\boldsymbol{y}) = p_i(\boldsymbol{y}) - L_{ijkl}(\boldsymbol{y})E_{kl}n_j$. Now, considering the first phase to be the hole and denoting

$\Gamma = \partial\Omega \cup \Gamma^C$, $s_i(\boldsymbol{y}) = -L_{ijkl}(\boldsymbol{y})E_{kl}n_j$ for $\boldsymbol{y} \in \partial\Omega$

and, $s_i(\boldsymbol{y}) = p_i^C(\boldsymbol{y})$ for $\boldsymbol{y} \in \Gamma^C$, (3) can be restored as:

$$c_{mn}(\xi)\hat{u}_n(\xi) = -\int_{\partial\Omega} p_{mi}^*(\boldsymbol{y},\xi)\hat{u}_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) + \int_{\partial\Omega} u_{mi}^*(\boldsymbol{y},\xi)p_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) + \int_{\Gamma} u_{mi}^*(\boldsymbol{y},\xi)s_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) \tag{4}$$

where the last term is given as a consequence of the unit strain loading. Differentiating (4) for ξ inside of the matrix gives:

$$\hat{\varepsilon}_m(\xi) = -\int_{\partial\Omega} P_{mi}^*(\boldsymbol{y},\xi)\hat{u}_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) + \int_{\partial\Omega} U_{mi}^*(\boldsymbol{y},\xi)p_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) + \int_{\Gamma} U_{mi}^*(\boldsymbol{y},\xi)s_i(\boldsymbol{y})\,\mathrm{d}\gamma(\boldsymbol{y}) \tag{5}$$

Discretization of (4) and (5) provides the concentration tensor:

$$\varepsilon_{ij}^{\mathrm{m}}(\boldsymbol{u}(\boldsymbol{y})) = A_{ijkl}^{\mathrm{m}}(\boldsymbol{y})E_{kl}, \quad \boldsymbol{y} \in \Omega^{\mathrm{m}} \tag{6}$$

Optimization

A natural question for engineers dealing with composites could be: determine such shape of fibers that the bearing capacity of the entire composite structure increases and attains maximum. This is a problem of optimal shape of structures and can be formulated for composites as follows: Let the uniform strain field E_{kl} be applied to the domain Ω (in our case, a periodic distribution of fibers is considered). This produces concentration factor A^{m}_{mnkl}, obeying (6). Let $\Pi(\boldsymbol{A}^{\mathrm{f}}, \boldsymbol{A}^{\mathrm{m}}, \Omega^{\mathrm{m}})$ be a real functional of A^{m}_{mnkl} and Ω^{m}. The problem of optimal shape consists of finding such a domain Ω^{f} from a class O of admissible domains, which minimizes Π. This may symbolically be written as

$$\text{Minimum } \{ \Pi(\boldsymbol{A}^{\mathrm{m}}, \Omega^{\mathrm{f}})\,;\ B(\boldsymbol{u}, \Omega^{\mathrm{f}}) = \boldsymbol{0} \} \qquad (7)$$

where B is an operator which for each Ω^{f} from O uniquely determines the displacement field $\boldsymbol{u}$ (in our case this is Eqs. (4)).

Such a problem may be formulated in terms of minimization of Lagranigian. In order to ensure the correctness of this formulation, additional constraints have to be applied. In our case, we assume the constant area of fibers (hole) and restriction of the distance of fiber from the boundary of the domain.

It remains to state the shape parameters $\boldsymbol{p}$ identifying the change of the boundary of the fibers. A natural choice is a movement of the boundary Γ^{C}. The Lagrangian involving the side condition using the Lagrangian multiplier λ is written as:

$$\Pi(\boldsymbol{u}, \Omega^{\mathrm{f}}) = \frac{1}{2} \int_{\Omega^{\mathrm{f}}} \sigma_{ij}(\boldsymbol{y}) \varepsilon_{\boldsymbol{ij}}(\boldsymbol{y})\, \mathrm{d}\Omega + \lambda\Big(\int_{\Omega^{\mathrm{f}}} \mathrm{d}\Omega^{\mathrm{f}} - C \Big) =$$

$$= \frac{1}{2} S_{ij} E_{ij} + \lambda\Big(\int_{\Omega^{\mathrm{f}}} \mathrm{d}\Omega^{\mathrm{f}} - C \Big) \qquad (8)$$

owing to Hill's energy condition. Substituting (6) to (8) gives:

$$\Pi(\boldsymbol{u}, \Omega^{\mathrm{f}}) = \frac{1}{2} [L^{\mathrm{m}}_{ijkl} < A(\boldsymbol{p})^{\mathrm{m}}_{kl\alpha\beta}(\boldsymbol{y}) >_{\mathrm{m}}] E_{ij} E_{\alpha\beta} + \\ + \lambda\Big(\int_{\Omega^{\mathrm{f}}} \mathrm{d}\Omega^{\mathrm{f}} - C \Big) \qquad (9)$$

and only the concentration factors are dependant of the vector $\boldsymbol{p}$. It appears that

$$\lambda = \frac{1}{2} [L^{\mathrm{m}}_{ijkl} < \frac{\partial A^{\mathrm{m}}_{kl\alpha\beta}(\boldsymbol{p})}{\partial p_{\boldsymbol{s}}} >_{\mathrm{m}}] E_{ij} E_{\alpha\beta} \Big/ \frac{\partial}{\partial p_s} \int_{\Omega^{\mathrm{f}}} \mathrm{d}\Omega^{\mathrm{f}} \qquad (10)$$

which means that the surface strain energy (10) is uniformly distributed along the interface, [1]. The internal parameters will be selected as rays from the origin of the symmetric unit cell. The optimal shapes for various volume ratios of the holes are shown in Fig. 1. In [1] affine transformation of the internal parameters is suggested, here the best approach leads to collinear transformation. Positively the symmetric shape is important for this decision.

Conclusions

The paper is focused on shape optimization of periodic composites, particularly on the composites with a hole. It appears that the choice of the cost functional leads to minimum of absolute value of both stresses and displacements, [1]. For simplicity, problem of optimal shape of hole is assumed and studies. The smaller the ratio, the shape is closer to a circle; the larger the ratio the shape converges to the square. Note that all directions are taken into account, which enables the localization and homogenization process proposed for composites by Suquet, who started with the weak formulation (distributions occur in the formulations). In this paper couple of arrangements has to be carried out to improve the formulation eligible for boundary elements.

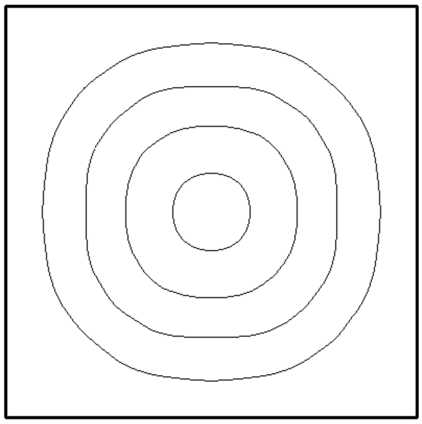

Fig. 1 Optimal shapes of holes for various hole ratio

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