ANALYSIS OF LARGE SYSTEMS USING LANCZOS VECTORS

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1. Introduction

Numerical Methods such as Finite Element Method, finer discritization of domain increases the accuracy of results. However FEM generates large numbers of linear equations in case of static or Dynamic analysis. Therefore solution of equations becomes difficult and time consuming due to large size of matrices. Also it becomes difficult to get the eigenvalues and eigenvectors in case of large and sparse matrices, especially when all the eigenvalues are not needed. The Lanczos algorithm for tridiagonalizing a symmetric matrix is the basis for several methods for solving such problems in static and dynamic analysis. A few examples are presented here.

2.1.1 Eigen Value Problems

The actual performance is illustrated by numerical examples [1].

2.2 SOLUTION OF LINEAR EQUATIONS

The Lanczos algorithm is also applicable in solving the linear equations[1], which is very common in static problems.

$$[K]\{X\} = \{F\} \qquad (1)$$

Right Hand Side may be zero in case of eigenvalue Problem

After performing a regular Lanczos step, multiplying Equation (1) by $[Q_l^T]$ and putting

$$\{X\} = [Q_l]\{U\} \qquad (2)$$

we get: $[Q_l^T]\{K\}[Q_l]\{U\} = [Q_l^T]\{F\}$ (3)

Or $[Tl]\{U\} = \{Pl\}$ (4)

Now the set of equations is very small in number since l< < n and above equations can be solved by any direct technique without any difficulty. After solving for {U}, the solution in original co-ordinates is obtained by multiplying {U} by $[Q_l]$ i.e.

$$\{X\} = [Ql]\{U\} \qquad (5)$$

Hence {X} is the solution of Equation (1).

3.6 EXAMPLES IN VIBRATION ANALYSIS

A few examples application of Lanczos mode superposition method (LMSM) in Dynamic analysis are presented below. Results are compared with the Classical Node Superposition Method (CMSM).

SIMPLE SPACE FRAME: The details are shown in Figure 1. The discrete modal comprises of 132 degrees of freedom

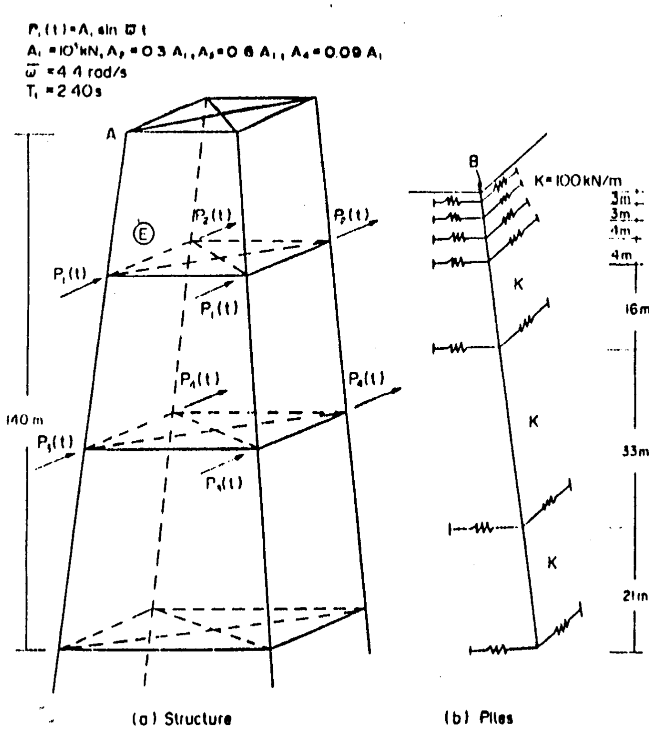

Fig. 1. Simple space frame characteristics.

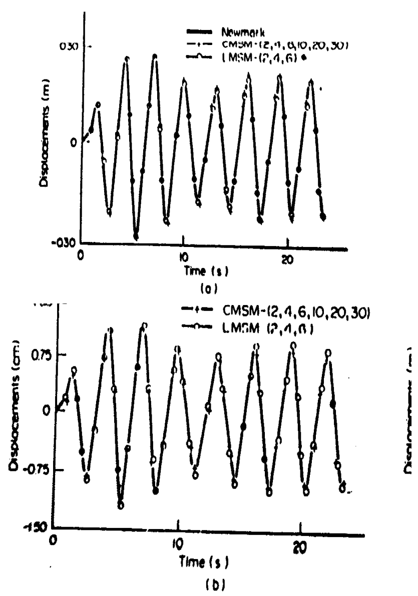

Fig. 2. Displacements' time-histories. (a) Displacements at node A. (b) Pile displacements (node B).

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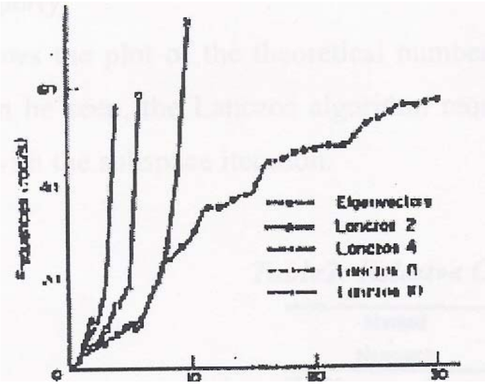


Fig. 3 Frequency Vs. Lanczos Approximations

Elastic Half Space: A semi infinite elastic half space subjected to a Ricker's wavelets in vertical and horizontal directions (Fig 6) are investigated.[1].

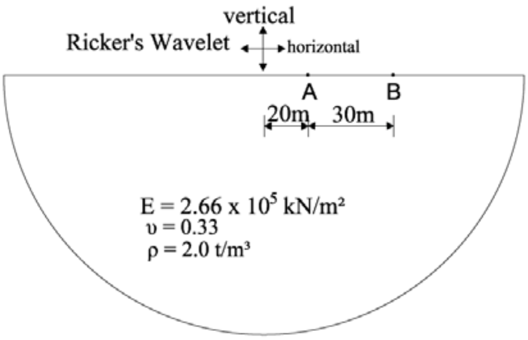


Fig. 4 Elastic Half Space Subjected to Ricker's wavelet

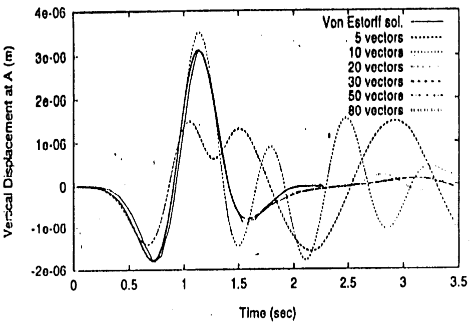


Fig. 5 No of Vectors on Displacement history (Vertical)

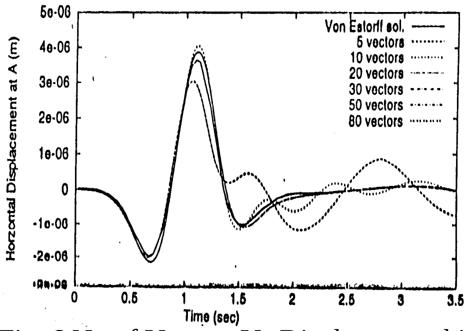


Fig. 6 No of Vectors Vs Displacement history (Horizontal)

Earthquake Analysis of Soil Structure Interaction

The model with vertical trenches is shown in Fig. 7 subjected to vertical and horizontal Base motion (Bhuj earthquake -India)

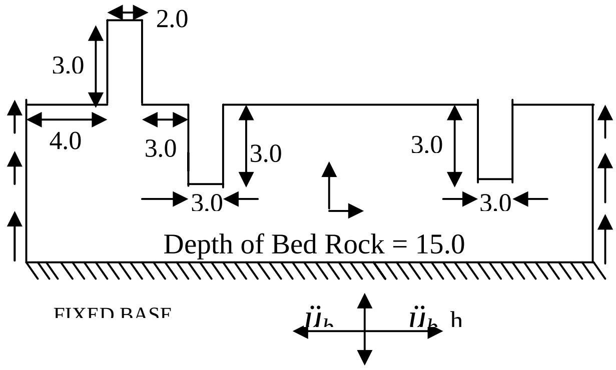
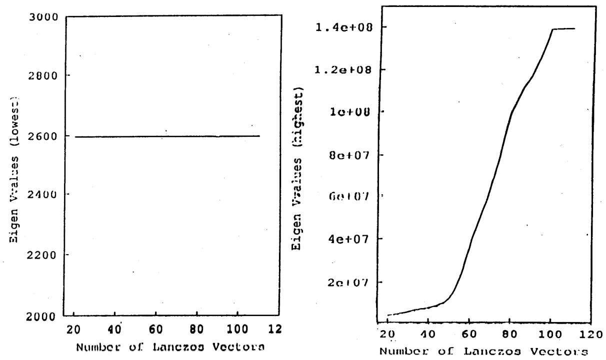


Fig. 7: Soil-Structure Interaction due to Earthquake



a.Lowest Eigenvalue b.Highest Eigenvalue

Fig. 8 Effect of No of Lanczos Vectors Vs Eigenvalues

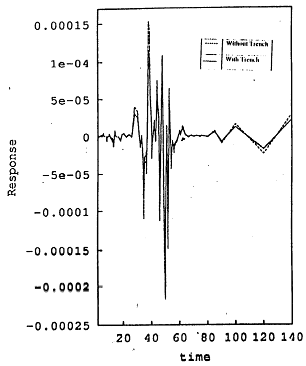


Fig. 9 Comparison of Displacements with and without trench (Isolation)

Reference: 1. KameswaraRao N.S.V and Ranjan Shrivastava, 'Analysis of Large Systems using Lanczos Method,' Research Report, Department of Civil Engineering, IIT, Kanpur, October, 2003.).