Hygrothermal Analysis of Rotating Composite Beams

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Introduction

The blade of a helicopter rotor, airplane propeller, or a wind turbine can be thought of as a rotating beam. The rotating natural frequencies of these blades affect the performance of the system. If such a blade is made of a polymer-matrix composite, operating environments such as moisture and high temperature may further influence the performance. The aim of this study is to analyze the effect of a hygrothermal condition on the free-vibration characteristics of a rotating beam made of polymer-matrix composite. The analysis consists of a theoretical formulation, finite element modeling using the *h-p* version, writing a computer code, and conducting a parametric analysis.

Theoretical Formulation

The theoretical formulation is accomplished in two major steps. The first step involves the use of Hamilton's principle to derive the equations of motion of a rotating composite beam incorporating first-order shear-deformation theory (FSDT). The second one deals with the empirical representation of the hygrothermal effects.

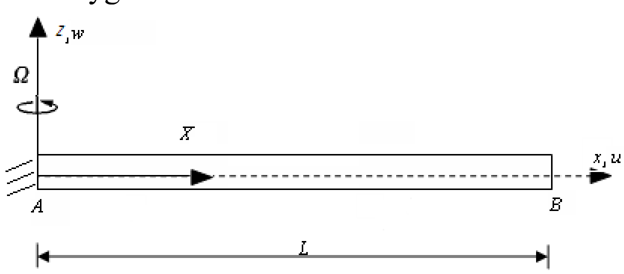


Fig. 1 Schematics of a rotating beam

Consider a beam *AB* of length *L* and mass per unit length *m* as shown in Fig. 1. The beam is cantilevered at point *A*, which is the origin for an inertial frame (*X,Y,Z*) with *X-Y* forming the plane of rotation and *Z*-axis normal to the this plane. The beam rotates with a constant angular velocity of Ω rad/s about the *Z*-axis. A moving coordinate frame (*x,y,z*) with its origin at *A* is attached to the beam and rotates with the beam. The beam undergoes an axial deformation, *u*, along the *x*-axis and a transverse deformation, *w*, along the *z*-axis.

In the present research, the reduction of composite plate equation to that of a beam is carried out in such a way as to retain significant influences of the beam characteristics and actions in the *y* direction. The composite plate constitutive equations for FSDT [1], after setting N_y, M_y, and Q_y to be zero are

$$\begin{Bmatrix} N_x \\ N_{xy} \\ M_x \\ M_{xy} \\ 0 \\ 0 \end{Bmatrix} = \left[\begin{array}{cccc|cc} A_{11} & A_{16} & B_{11} & B_{16} & A_{12} & B_{12} \\ A_{16} & A_{66} & B_{16} & B_{66} & A_{26} & B_{26} \\ B_{11} & B_{16} & D_{11} & D_{16} & B_{12} & D_{12} \\ B_{16} & B_{66} & D_{16} & D_{66} & B_{26} & D_{26} \\ \hline A_{12} & A_{26} & B_{12} & B_{26} & A_{22} & B_{22} \\ B_{12} & B_{26} & D_{12} & D_{26} & B_{22} & D_{22} \end{array}\right] \begin{Bmatrix} \varepsilon_x^{(0)} \\ \gamma_{xy}^{(0)} \\ \varepsilon_x^{(1)} \\ \gamma_{xy}^{(1)} \\ \varepsilon_y^{(0)} \\ \varepsilon_y^{(1)} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_x \\ 0 \end{Bmatrix} = K \begin{bmatrix} A_{55} & A_{45} \\ A_{45} & A_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix}$$

The governing equations of the problem is derived from Hamilton's principle,

$$\Delta = \int_{t_1}^{t_2} \left(\delta U - \delta T - \delta W \right) dt = 0$$

where δU is the virtual strain energy, δT is the virtual kinetic energy, δW is the virtual work and t is the time coordinate. For a free-vibration analysis $\delta W = 0$ while the expressions for δU *and* δT are

$$\delta U = b\int_0^L \left[N_x \left(\delta u_0' + w_b' \delta w_b' + w_s' \delta w_s' \right) - M_x \delta w''_b + N_{xy} \delta \gamma_0 - 2M_{xy} \delta w_b'^{y} + Q_x w_s' \right] dx$$

$$\delta T = \iiint_V \rho \left[\left(\dot{u}_0 - z\dot{w}_b' \right) \delta \dot{u}_0 - \left(z\dot{u}_0 - z^2 \dot{w}_b' \right) \delta \dot{w}_b' + \left(z^2 \dot{w}_b^{y} \right) \delta \dot{w}_b^{y} + \left(\dot{w}_b + \dot{w}_s \right) \delta \dot{w}_b + \left(\dot{w}_b + \dot{w}_s \right) \delta \dot{w}_s \right] dV$$

The analysis of composite laminates under the effect of hygrothermal exposure can be done in two ways. The first one considers the residual strains caused due to different expansion rates between the matrix and fiber. The second approach degrades the strength and stiffness related properties of the constituents. The present research considers the second effect. According to Chamis [2], the relationship between the wet resin and dry resin mechanical properties are given as

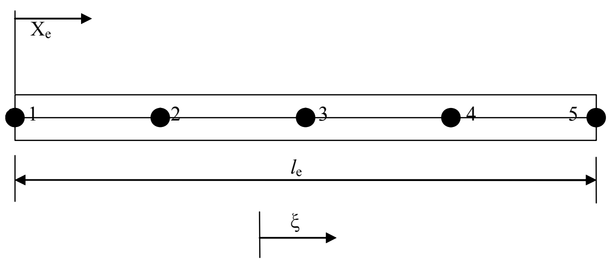
$$\frac{P_{HTM}}{P_0} = \left[\frac{T_{gwr} - T}{T_{gdr} - T_0}\right]^{0.5}$$

where P is the property to be measured, HTM refers to hygrothermal mechanical condition, T_{gwr} is wet resin glass transition temperature and T_{gdr} the dry resin glass transition temperature. T_0 is room temperature and T is the temperature at which the property is to be measured.

Finite Element Formulation

The finite element used is a custom-designed *h-p* version, 5-node, 29-d.of. beam element as shown in Fig. 2. The detailed derivation of the element stiffness and inertia matrices from Hamilton's principle indicated above can be seen in Ref. 3. After assembling the element matrices, the global equations of motion is written as

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\}$$



DOF at end nodes (1, 5): $u, \gamma, w_b, w_b', w_s, w_s', w_b^y$

DOF at internal nodes (2, 3, 4): $u, \gamma, w_b, w_s, w_b^y$

Fig.2 An element with nodes and d.o.f.

A computer program is written in MATLAB® to carry out the finite element modeling and solution. The Gauss quadrature is used to numerically calculate the element matrices.

Results and Discussions

It is assumed that only the matrix properties are affected by the temperature and moisture changes. The Chamis equation is used to calculate the reduced matrix properties and then the rule of mixtures is applied to compute the reduced ply properties E_1, E_2, G_{12}, and ν_{12}. The beam considered is an S-Glass/Epoxy laminate. A parametric study is conducted by varying temperature, moisture content, ply-orientation and rotating speed. The dimensions of the beam considered in the present study are length (L) = 0.190 m, width (b) = 0.0127 m, height (h) = 0.003175 m. All the results presented correspond to a fiber volume fraction V_f = 0.6 and the temperature for dry condition (m = 0%) is taken to be 21 ^0C. For the hygrothermal condition two cases of temperature 52 ^0C and 90 ^0C are considered. The material properties used in this analysis are listed in Ref. 3. The verification of the dynamic and hygrothermal models is achieved by selecting appropriate models [3].

The results shown in Fig. 3 are rotating natural frequencies (under dry and 5% moisture conditions) of the first four bending modes (1B etc.) as a function of the rotational speed of a 24-ply 15-degree angle-ply beam. It is seen that the effect of a hygrothermal environment is more pronounced for higher modes. The effect decreases as the rotational speed increases; the reason for this is the decrease in material stiffness due to hygrothermal effect is partially offset by the increase in beam stiffness due to the centrifugal effect of the rotational speed.

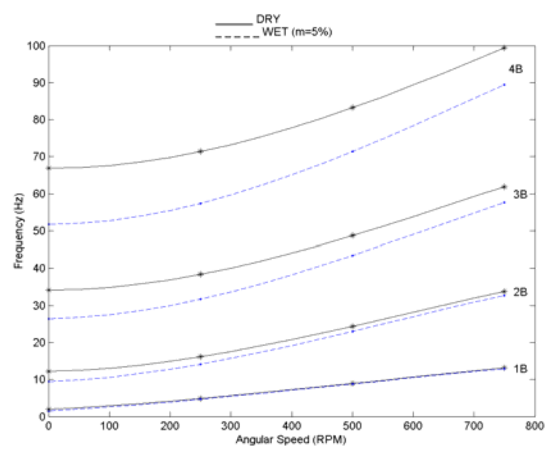


Fig. 3 Rotating natural frequencies of composite beam, $[15]_{24}$, T=**90** ^0C

References

1. Reddy, J.N., (1997), "*Mechanics of Laminated Composite Plates: Theory and Analysis*", CRC Press, Boca Raton, Fla.
2. Chamis, C.C., (1983), "Simplified Composite Micromechanics Equations for Hygral, Thermal and Mechanical Properties", NASA Technical Memorandum 83320.
3. Vennam, S., (2006), "Hygrothermal Effects on free Vibration Characteristics of Rotating Composite Beams," MSME Thesis, West Virginia University, Morgantown, WV.