Use the Crack 3D Code with Ansys Software to Solve the Composite Cylinder with Crack Torsion Problem

Xinyan Tang [a,1], Xiaomin Deng [b,*], Michael A. Sutton [b]

[a] *Department of Mechanical Engineering, Engineering College of Nanjing Agricultural University, Nanjing, Jiangsu, 210031, China, txy32@tom.com*

[b] *Department of Mechanical Engineering, University of South Carolina, Columbia SC 29208 USA*

1. the Composite Cylinder Torsion Problem;

As shown in Figure 1, let Ω be the general cross-section of a cylinder, Γ be the boundary of the cross-section Ω and (x, y, z) be the principal axes of inertia. Though not shown the oz axis and the origin o lie at centroid c of the lower end cross-section, i.e. the bottom section Ω_b. The torsion moment M_k is applied on the upper end cross-section, i.e. the top section Ω_t.

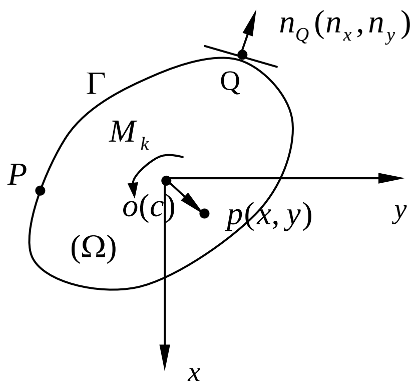

Fig. 1 general cross-section of a torsion cylinder

The length of the cylinder is l. In rectangular coordinates (x, y), the displacement components of the cylinder applied by torsion moment M_k can be expressed as follows[1]

$$u_x = -\alpha\, z\, y, \qquad u_y = \alpha\, z\, x,$$

$$u_z = \alpha\, \varphi(x, y) \qquad (1)$$

where α is the twisting angle per unit length of the cylinder, i.e. twisting ratio, which can be determined by the relative angle of rotation between two different cross-sections. Such as the top section Ω_t and bottom section Ω_b of the cylinder. Letting $\varphi(x, y)$ be a plane harmonic function in region Ω, the torsion rigidity D is determined using the following area integral

$$D = \frac{M_k}{\alpha} = \mu \int_{\Omega} \left(x \frac{\partial \varphi}{\partial y} - y \frac{\partial \varphi}{\partial x} + x^2 + y^2 \right) d\Omega \quad (2)$$

in which $\mu = \dfrac{E}{2(1+\nu)}$ is the shearing modulus, M_k is determined by the external load applied to the upper end cross-section of the cylinder. The corresponding torsion shearing stresses (τ_{zx}, τ_{zy}) on the cross-section Ω are given by

---

[1] As a visiting scholar in the Department of Mechanical Engineering, University of South Carolina, Columbia SC 29208 USA

* Corresponding author. Tel.: +1-803-777-7144; fax: +1-803-777-0106.
E-mail address: deng@engr.sc.edu (X. Deng).

$$\tau_{zx}(x,y)=\alpha\mu\left[\frac{\partial\varphi}{\partial x}-y\right]$$

$$\tau_{zy}(x,y)=\alpha\mu\left[\frac{\partial\varphi}{\partial y}+x\right] \qquad (3)$$

2. Crack3d finite element method to solve Saint-Venant torsion problem

To the author's knowledge, the boundary element method[2] has been used extensively to solve the Saint-Venant torsion problem in elasticity. In solid mechanics, the finite element method[3] (FEM) is a well known, versatile, flexible numerical method that is being used extensively in almost all fields of science and engineering. Since it does not appear that the FEM has been used to computationally solve the Saint-Venant torsion problem, one of the goals of this paper is to outline a combination of theory and FEM analyses to solve an equivalent Saint-Venant torsion cylinder. Here the ***CRACK3D*** code with ***ANSYS*** software finite element method, developed by Drs. Jianzheng Zuo, Xiaomin Deng, and Michael A, Sutton[4], will be used to solve the general torsion cylinder in order to get the relative numerical results on the boundary Γ equivalent to the Saint-Venant torsion solution. After substituting these results into the boundary element discretion formula derived from Saint-Venant torsion theory, the torsion numerical solution for the equivalent Saint-Venant cylinder torsion problem is obtained.

3. Numerical results of Saint-Venant torsion problem with crack

We consider a equivalent Saint-Venant torsion problem on a general cylinder($20\times 20\times 60mm$) contains an edge crack. Using the method of ***CRACK3D*** Code with ***ANSYS*** software to calculate this problem. The results are given in below table:

.

α (^{0}/mm)	2.4523E-5
$\frac{D}{\mu a^4}$	1.1051
$\frac{\tau}{\alpha\mu a}$	1.0783
$\frac{M_k}{\tau a^3}$	1.1849
K_{III} (N/mm$^{3/2}$)	11.6780

Acknowledgement

The financial aid is provided by the China Scholarship Council. The academic support is provided by the Department of Mechanical Engineering at the University of South Carolina in Columbia, SC.

References

[1] Muskhelishvili, Some Basic Problems of the Mathematical Theory of Elasticity, Netherlands,1953.

[2] Brebbia,C.A., Boundary Element Techniques, Springger-Verlag, Berlin,1984.

[3] Zienkiewicz O.P., The Finite Element Method in Engineering Science, London,1971.

[4] Jianzheng Zuo, Xiaomin Deng and Michael A. Sutton. User's Manual and Verifications in CRACK3D version 4.50, University of South Carolina, 2007.