PIEZOELECTRIC VOLTAGE EVALUATION OF MEMS ENERGY HARVESTER WITH INTERDIGITATED ELECTRODES

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A method for evaluation of piezoelectric voltage of vibrating MEMS energy harvester is considered in the paper. The vibrating element is a bimorph cantilever with a piezoelectric layer. The piezoelectric voltage arises as a result of bending stress of the vibrating cantilever. Interdigitated electrodes are used in order to increase the efficiency of the energy harvester.

Keywords: *Energy harvester, MEMS, vibration, interdigitated electrodes.*

1 Introduction

There are plenty of vibration energy sources – rotating objects, subways, pipelines, HVAC systems, and even the human body. Roundy [2005] presents basic equations for using the piezoelectric effect and other types of coupling mechanical to electrical elements. The paper also states that for any harvester, the power output depends on the system's coupling coefficient, the dynamic quality factor, density of the generator, and electrical load resistance. Roundy explains also that piezoelectric materials with large coupling coefficients are best for harvesting.

The piezoelectric materials are anisotropic and there are two common modes utilized for piezoelectric energy harvesting: 33 - mode (stack actuators) and 31 - mode (bimorphs) [Priya et al. 2009]. In 33 - mode, the direction of applied stress (force) and generated voltage is the same, while in 31 - mode the stress is applied in axial direction but the voltage is obtained from perpendicular direction.

Since most of the piezoelectric materials exhibit large permittivity, in order to avoid the additional capacitance and its electrostatic energy, the electrodes of interdigitated structure are the most suitable in 33 mode. Kim [2008] has demonstrated piezoelectric mirco-cantilevers-based energy harvester utilizing interdigital electrode pattern to access the d_{33} response.

The present paper considers a method for evaluation of piezoelectric voltage of bimorph cantilever with interdigitated electrodes (IDE).

2 Description of the method

The free body diagram of the cantilever, the bending moment M_y, and the top view of the cantilever with interdigitated electrodes is shown in Figure 1. The voltage between two teeth can be calculated by [Jeon et al. 2005], [Choi et al. 2006] the formula

(1) $$V_i = \sigma_{xxm_i} t g_{33},$$

where σ_{xxm_i} is the mean normal tension between the $(i-1)^{\text{th}}$ and the i^{th} electrode, t is the step, g_{33} is the piezoelectric constant. Since the circuit of interdigitated electrodes provides serial connection the total voltage is

(2) $$V_i = \sum_{i=1}^{n} V_i = t g_{33} \sum_{i=1}^{n} \sigma_{xxm_i}.$$

The mean normal tension here is assumed to be

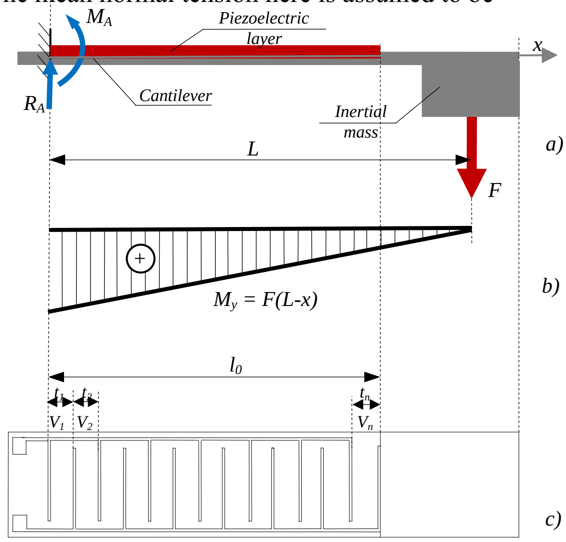

Figure 1. Parameters of piezoelectric sensor: a) free body diagram; b) bending moment diagram; c) top view of the cantilever

(3) $$\sigma_{xxm_i} = \frac{\sigma_{xx_i} - \sigma_{xx_{i-1}}}{2}.$$

Since the cantilever is bended the normal tension can be presented at the form

(4) $$\sigma_{xx_i} = \frac{M_y}{W_y},$$

where the bending moment is

(5) $$M_y = F\left(L - x\right)$$

and the section modulus is

(6) $$W_y = \frac{wh^2}{6}.$$

Here w is the width and h is the high of the cross section of the cantilever.

Neglecting piezoelectric layer thickness the static deflection of the free end of the cantilever can be expressed approximately by the formula

(7) $$y = \frac{L^3}{3EI} F,$$

where L is the length of the cantilever, E is Young modulus, the moment of inertia of cross section is

(8) $$I = \frac{wh^3}{12}.$$

The force F is expressed from (7) and substituted in formula (5). So for the bending moment it is obtained

(9) $$M_y = \frac{3EI}{L^3}(L - x)\, y.$$

By substituting (9) in (4) the normal tension

(10) $$\sigma_{xx} = \frac{3EI}{L^3 W_y}(L - x)\, y = \frac{3Ew}{2L^3}(L - x)\, y$$

is found. For the i^{th} tooth the stress is

(11) $$\sigma_{xx_i} = \frac{3Ew}{2L^3}(L - it)\, y.$$

Taking in account (3) and (11) after simple transformation the mean tension is found in the view

(12) $$\sigma_{xxm_i} = \frac{3Ew}{2L^3}\left(L - it + \frac{t}{2}\right) y.$$

The so obtained tension of formula (12) is substituted in (1) and it is yield

(13) $$V_i = \frac{3Ewg_{33}}{2L^3}\left(L - it + \frac{t}{2}\right) ty.$$

According to formula (2) the total piezoelectric voltage is

(14) $$V = \frac{3Ewg_{33}y}{2L^3}\sum_{i=1}^{n}\left[\left(L - it + \frac{t}{2}\right) t\right] = \frac{3Ewg_{33}y}{2L^3}\left(Ltn - \frac{1}{2}t^2 n^2\right).$$

Since the length of the piezoelectric layer l_0 according to Figure 3 is

(15) $$l_0 = tn.$$

The total voltage takes the form

(16) $$V = \frac{3Ewg_{33}y}{2L^3}\left(Ll_0 - \frac{1}{2}l_0^2\right).$$

After denoting

(17) $$d_p = \frac{3Ewg_{33}}{2L^3}\left(Ll_0 - \frac{1}{2}l_0^2\right)$$

formula (16) is rewritten in the form

(18) $$V = d_p y.$$

3 Determining the optimal length of the piezoelectric layer

The coefficient d_p from the expression (17) can be used for estimation of the optimal length of the piezoelectric layer. For example if it is denoted

(19) $$\lambda_v = \frac{l_0}{L}$$

and it follows

(20) $$d_p = \frac{3Ewg_{33}}{2L}\left(\lambda_v - \frac{1}{2}\lambda_v^2\right).$$

The function

(21) $$\Lambda = \lambda_v - \frac{1}{2}\lambda_v^2$$

obtains its maximum $\Lambda_{\max} = 1/2$ for $\lambda_v = 1$. If we assume to cover only 50% of L with piezoelectric layer or $\lambda_v = 1/2$ the ratio of the obtained voltage in this case to the maximum possible one is $\Lambda/\Lambda\max = 2\left(\lambda_v - \lambda_v^2/2\right) = 1 - 0.25 = 0.75$. This result shows that the voltage near the free end is weak and it is not necessary to cover the whole length L with piezoelectric layer.

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5 Conclusions

The so presented method has some disadvantages that could lead to certain inaccuracies. The main reason for these inaccuracies is the preliminary assumed simplifications. The simplified assumptions are: the static deformation of the cantilever is used instead of the possible dynamic deformation modes; the calculations are made under consideration of small beam deflections, which are typical for macro devices. It is known that MEMS devices possess large deflections and the linear theory is not suitable enough according to Ginsberg [2005].

The above mentioned disadvantages limit the applications of the method. Its applicability is suitable for initial rough calculations in lumped parameter models.

References

1. Roundy, Shad. On the effectiveness of vibration-based energy harvesting. *Journal of Intelligent Material Systems and Structures*, Vol. 16, No 10, 2005, pp. 809-823.
2. Priya S., Inman D. J. Energy Harvesting Technologies, *Springer Science* + *BusinessMedia*, LLC, 2009, p. 522.
3. Choi, W.J., Jeon Y., Jeong J.H. Sood R. Kim S.G. Energy harvesting MEMS device based on thin film piezoelectric, *J Electroceram, Springer Science + Business Media*, LLC, 2006 pp. 543-548
4. Jeon Y.B., Sood R., Jeong J.H. and Kim S.-G. MEMS power generator with transverse mode thin film PZT, *Sensors and Actuators A: Physical*, Volume 122, Issue 1, *Elsevier*, 2005, pp. 16-22
5. Kim, H, Bedekar, V, Islam, R, Lee, WH, Leo, D, and Priya, S. Laser micro-machined piezoelectric cantilevers for mechanical energy harvesting. *IEEE Ultrason. Freq. Ferroelect. Cntrl*, 2008, pp. 1900–1905.
6. Ginsberg J. H. Advanced Engineering Dynamics, *Cambridge University Press*, 2005, pp. 18-20.