In-plane Fracture Analysis of an Embedded Central Crack in Functionally Graded Strip Bonded by Two Homogeneous Half Planes

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Abstract

This paper investigates the in-plane fracture problem of an embedded central crack in the FGM strip bonded by two dissimilar homogeneous half planes under in-plane loads. The material gradient is assumed to be in an exponential form. The effects of the crack length and the inhomogeneous material parameter on stress intensity factors and the strain energy density factor are discussed.

Keywords: Crack; Functionally graded material; Singular integral equation; In-plane; Stress intensity factor; Strain energy density factor.

1. INTRODUCTION

The materials are required to be versatile because of the severe environments such as high temperatures and corrosion in modern engineering applications. Therefore, a variety of metal/ceramic composites and ceramic thermal barrier coatings (TBCs) have been developed. However, there are some disadvantages due to the highly-different material properties between the coating and the substrate such as high thermal stresses, residual stresses, and relatively poor bonding strength. Materials may have an interfacial failure because of these disadvantages. One way to reduce the difference of material properties from different materials is to introduce Functionally Graded Materials (FGMs). FGMs are multiphase composite with continuously varying volume fractions as shown in Fig. 1.

The in-plane crack problems for a FGM medium were studied by Delale and Erdogan [1]. Chen and Erdogan [2] studied an interface crack between an inhomogeneous strip, whose properties varied along the direction perpendicular to the crack, and an infinite homogeneous plane under in-plane loads. Shbeeb and Binienda [3] studied an interface crack problem while the structure consisted of a homogeneous material, a FGM strip, and another homogeneous material.

The main objective of this paper is to study the fracture problem of a central crack of length 2a embedded in the FGM strip bonded by two dissimilar elastic half planes subjected to in-plane mechanical loads. The material gradient is to be assumed to be in an exponential function and perpendicular to the crack. Numerical results of stress intensity factors and the strain energy density factor are shown to discuss the effects of the length of the crack and the inhomogeneous parameter.

Fig.1 Continuously varying function.

2. FORMULATION OF THE PROBLEM

The geometry of the problem is shown in Fig. 2.

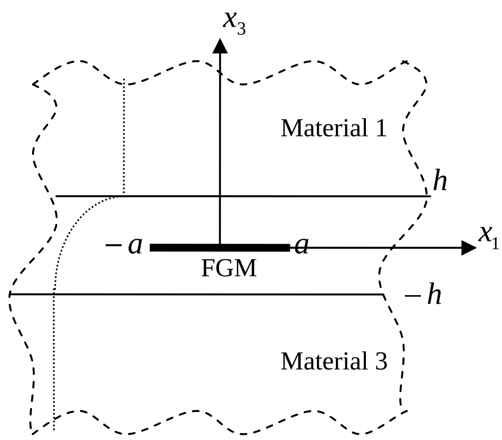

Fig. 2 Geometry of the problem.

The material properties are assumed to be in the following exponential forms:

$$C_{11}^{(2)}(x_3) = C_{110}e^{\beta x_3},\ C_{12}^{(2)}(x_3) = C_{120}e^{\beta x_3} \tag{1}$$

where $C_{ij}^{(2)}$ represent material properties in the strip and C_{110} and C_{120} denote the stiffness constants at $x_3 = 0$.

From governing equations and static equilibrium equations, the displacements and stresses can be obtained by applying the Fourier transform with respect to x_1. The total of 12 undetermined functions in displacements and stresses can be solved with the boundary conditions and dislocation density functions.

3. SINGULAR INTEGRAL EUQATIONS

Since the dislocation density functions are used to solve these unknowns, the problem can be formulated as a system of singular integral equations. The following equations can be obtained:

$$p(x_1)=\frac{1}{2\pi}\int_{-a}^{a}\sum_{j=1}^{2}M_{1j}f_j(t)dt\,,$$

$$q(x_1)=\frac{1}{4\pi}\int_{-a}^{a}\sum_{j=1}^{2}M_{2j}f_j(t)dt \tag{2}$$

where $p(x_1)$ and $q(x_1)$ are external normal load and shear load. After separating the singular terms, the kernels can be written as:

$$M_{ij}(\alpha)=2Nr_{ij}(\alpha)+2\frac{Ns_{ij}}{t-x_1} \tag{3}$$

where $Nr_{ij}(\alpha)$ are functions of α and Ns_{ij} are two real constants.

From the definition, stress intensity factors can be expressed as:

$$k_1(a)=\frac{-1}{\sqrt{a}}Ns_{12}F_2(a),\ k_2(a)=\frac{-1}{2\sqrt{a}}Ns_{21}F_1(a) \tag{4}$$

4. Numerical procedures

By applying the Gauss-Chebyshev integration formula, we have the following algebraic equations to solve $F_j(t_n)$:

$$\sum_{k=1}^{n}\left[\frac{Nr_{11}(ax_r,at_k)}{n}F_1(t_k)+\frac{Nr_{12}(ax_r,at_k)}{n}F_2(t_k)+\frac{Ns_{12}}{an(t_k-x_r)}F_2(t_k)\right]=-p(ax_r),$$

$$\frac{1}{2}\sum_{k=1}^{n}\left[\frac{Nr_{21}(ax_r,at_k)}{n}F_1(t_k)+\frac{Ns_{21}}{an(t_k-x_r)}F_1(t_k)+\frac{Nr_{22}(ax_r,at_k)}{n}F_2(t_k)\right]=-q(ax_r),$$

$$\sum_{k=1}^{n}\frac{\pi}{n}F_1(t_k)=0,\ \sum_{k=1}^{n}\frac{\pi}{n}F_2(t_k)=0 \tag{5}$$

Since the singular integral equations were solved, stress intensity factors can be rewritten as:

$$k_1(a)=\frac{-1}{\sqrt{a}}Ns_{12}F_2(1),\ k_2(a)=\frac{-1}{2\sqrt{a}}Ns_{21}F_1(1) \tag{6}$$

The strain energy density factor at crack tip a also can be obtained as:

$$S_a(\theta)=\frac{1}{16\left(C_{110}^2-C_{120}^2\right)}\left\{\begin{array}{l}C_{110}\left[a_{11}k_1^2(a)+2a_{12}k_1(a)k_2(a)+a_{13}k_2^2(a)\right]\\+C_{120}\left[a_{21}k_1^2(a)+2a_{22}k_1(a)k_2(a)+a_{23}k_2^2(a)\right]\end{array}\right\} \tag{7}$$

where a_{ij} are:

$$a_{11}=5+4\cos\theta-\cos 2\theta\ ,\quad a_{12}=2(-2\sin\theta+\sin 2\theta)\ ,$$

$$a_{13}=9-4\cos\theta+3\cos 2\theta\ ,\quad a_{21}=-4\cos^2\frac{\theta}{2}(1+\cos\theta)\ ,$$

$$a_{22}=8\cos^2\frac{\theta}{2}\sin\theta\ ,\quad \text{and}\quad a_{23}=-4\cos^2\frac{\theta}{2}(1-3\cos\theta)$$

with θ being the angle counter clockwise measured from x_1-direction at crack tip a.

5. Results and discussions

If the structure is a homogeneous medium, i.e., the values of βh go to zero, the values of $k_1(a)/\sqrt{a}$ should approach to $p_0=3MPa$. It is because $k_1(a)=p_0\sqrt{a}$ when the crack is in a homogeneous medium. Table 1 shows the variations of the values of $k_1(a)/\sqrt{a}$ with various values of βh at $a/h=1.2$.

In Fig. 3, the angle of $S_{\min}$ decreases to negative values from 0° with increasing values of βh . It means that the angle of $S_{\min}$ follows the direction with softer material properties.

Table 1 Values of $k_1(a)/\sqrt{a}$ with various βh .

βh	$k_1(a)/\sqrt{a}(MPa)$
1	2.911862146
0.75	2.956260147
0.5	2.979195618
0.1	2.999188475
0.05	2.999798644

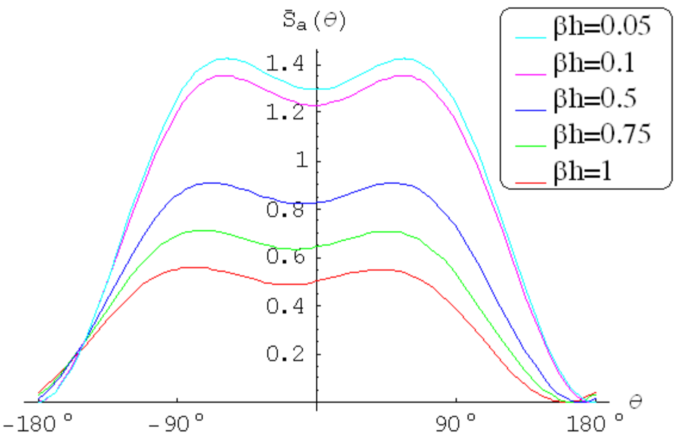

Fig. 3 Variations of $\bar{S}_a(\theta)$ with θ at various βh .

6. Conclusions

The angle of $S_{\min}$ follows the direction with softer material properties. Therefore, we boldly suppose that it is still proper for inhomogeneous materials that the crack initiation occurs at the angle of $S_{\min}$.

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