VISCOELASTIC ANALYSIS OF FRP STRENGTHENED CONCRETE BEAMS

Jialai Wang, **Chao Zhang**
Deptt. of Civil, Const., and Environ. Engineering, The University of Alabama, Tuscaloosa, AL 35487

INTRODUCTION

Externally bonding of FRP plates or sheets has become a popular method for strengthening reinforced concrete structures. Stresses along the FRP-concrete interface are of great importance to the effectiveness of this type of strengthening because high stress concentration along the FRP-concrete interface can lead to the FRP debonding from the concrete beam. Although extensive studies have been done on the short-term stresses of the FRP-concrete interface, very few studies have been conducted on the long-term stress distribution along the FRP-concrete interface. In this study, we develop a viscoelastic solution for the interface stresses in a FRP plate strengthened RC beam.

VISCOELASTIC MODEL

Consider a simply supported RC beam (beam 1) with thickness of h_1 strengthened by a thin FRP plate (beam 2) with thickness of h_2 through external bonding with a thin adhesive layer with thickness of h_0, as shown in Fig. 1. A uniform load with magnitude of q is applied to the RC beam. Only the adhesive layer is modeled as viscoelastic material, and the time-dependent properties of FRP plate and RC beamare ignored in this study for the sake of simplicity.

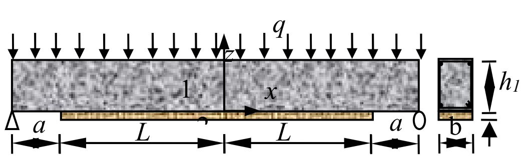

Fig.1--A FRP strengthened RC beam

Both the RC beam and the FRP plate are modeled as Timoshenko beams, beam 1 and 2, respectively. The displacement fields can then be written as

$$U_1(x_1,z_1,t)=u_1(x_1,t)+z_1\phi_1(x_1,t);$$
$$U_2(x_2,z_2,t)=u_2(x_2,t)+z_2\phi_2(x_2,t)-u_0(x_2,t)$$
$$W_i(x_i,z_i,t)=w_i(x_i,t) \qquad (1)$$

where subscript i =*1,2*, representing the beams *1* and *2* in Fig.1. $u_i(x, t,)$, $\phi_i(x, t)$ and $w_i(x, t,)$ are the axial displacement, rotation and deflection at the neutral axis of beam i. $u_0(x,t)$ is the initial displacement of the FRP plate before the pretension is released. $U_i(x_i, z_i, t)$ and $W_i(x_i, z_i, t)$ are the axial and transverse displacements of beam i.

Considering an infinitesimal free body diagram of the FRP plate strengthened RC beam shown in fig. 2, the following equilibrium equations can be established

$$\frac{\partial N_1(x,t)}{\partial x}=b\,\tau(x,t),\quad \frac{\partial M_1(x,t)}{\partial x}=Q_1(x,t)-\frac{h_1}{2}b\,\tau(x,t),$$
$$\frac{\partial Q_1(x,t)}{\partial x}=b\sigma(x,t)+bqH(t),$$
$$\frac{\partial N_2(x,t)}{\partial x}=-b\,\tau(x,t),\quad \frac{\partial M_2(x,t)}{\partial x}=Q_2(x,t)-\frac{h_2}{2}b\,\tau(x,t),$$
$$\frac{\partial Q_2(x,t)}{\partial x}=-b\sigma(x,t). \qquad (2)$$

where $\tau(x,t)$ and $\sigma(x,t)$ are the interface shear and normal stress within the adhesive layer, respectively.

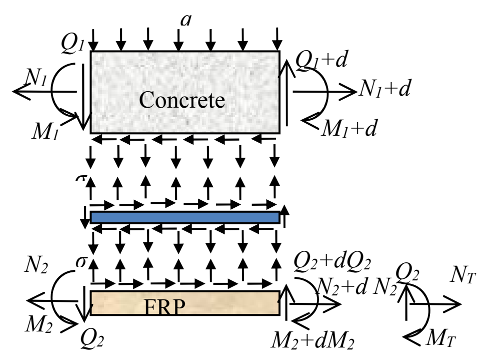

Fig. 2--Free body diagram

Then the average strains within the adhesive layer can be established as (Delale and Erdogan 1981)

$$\gamma_{xy}(x,t)=\left(u_0(x,t)+u_1(x,t)-\frac{h_1}{2}\phi_1(x,t)-u_2(x,t)-\frac{h_2}{2}\phi_2(x,t)\right)/h_0$$
$$\varepsilon_y(x,t)=\left(w_1(x,t)-w_2(x,t)\right)/h_0\ ,$$
$$\varepsilon_x(x,t)=\left(\frac{\partial u_1(x,t)}{\partial x}-\frac{h_1}{2}\frac{\partial\phi_1(x,t)}{\partial x}+\frac{\partial u_2(x,t)}{\partial x}+\frac{h_2}{2}\frac{\partial\phi_2(x,t)}{\partial x}-\frac{\partial u_0(x,t)}{\partial x}\right)/2 \quad (3)$$

The constitutive equations of linear isotropic viscoelastic materials can be expressed by means of differential operators in following forms

$$P_1(s_{ij})=Q_1(e_{ij}),\quad (i,j)=1,2,3\ , \qquad (4a)$$
$$P_2(s)=Q_2(e)\ , \qquad (4b)$$

where P_1, Q_1, P_2 and Q_2 are differential operators; s_{ij} and e_{ij} (i, j = 1,2,3) are the deviatoric components of the stress and strain tensors, respectively; e and s are the volumetric strain and hydrostatic component of stress tensor, respectively.A Standard Linear Solid (SLS) is employed in this study, as shown in Fig. 3. Then the differential operator of the adhesive can be expressed as

$$P_1 = a_1 + \frac{d}{dt}, \quad Q_1 = b_1 + b_2 \frac{d}{dt} \;, \qquad (5)$$

where, $a_1 = \frac{k_2}{\eta} = \frac{1}{t_0}$; $b_1 = \frac{k_1 k_2}{\eta} = \frac{k_1}{t_0}$; $b_2 = k_1 + k_2$; and t_0 is the retardation time.

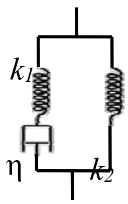

Fig. 3--Standard Linear Solid Model

VALIDATION

A simply supported RC beam strengthened by an FRP plate shown in Fig.1 is studied using the present analytical method and finite element analysis. As shown in Fig.1, a uniformly distributed load q = 0.1 N/mm^2 is applied to the RC beam. The geometry of the structure shown in Fig. 1 is given as: a = 300 mm, L = 1200 mm, h_1 = 300 mm, h_2 = 4mm, h_0 = 2 mm, and b = 200 mm. The material properties are given as E_1 = 30GPa, v_1 =0.18, E_2 = 100GPa, v_2 = 0.35, E_a = 2GPa, v_a = 0.3. G_a/G_∞ = 3, and t_0 = 5 days.

Fig. 4and 5 show the interface shear and normal stresses obtained by the present method and FEA for the FRP strengthened RC beams shown in Fig. 1. It can be observed that the present model agrees very well with FEA except a small zone near the free edge of the adhesive layer. In this small zone, FEA shows that shear stress reduces to zero at the free edge; while the present analytical model shows that the shear stress reaches its maximum at the free edge. This discrepancy is caused by the inherent shortcoming of G-R model of the adhesive layer, which is essentially a two-parameter elastic foundation model. As illustrated in details by Wang and Zhang(2010), the two-parameter elastic foundation model of the adhesive layer can't satisfy the zero-shear stress boundary condition at the free edge of the adhesive layer. To overcome this shortcoming, a three-parameter elastic foundation model proposed by Wang and Zhang (2010) should be used. As anticipated, significant interface stress redistributions with time are observed from both figures. Both the shear and normal interface stress concentrations are alleviated due to the creep deformation of the adhesive layer.

Fig. 4--Redistribution of interface shear stress with time (days)

Fig. 5--Redistribution of interface normal stress with time (days)

ACKNOWLEDGE

This study was supported by the National Science Foundation (Grant No.: CMMI-0927938 under program director Dr. ***Lawrence C. Bank***).

REFERENCES

Delale F., Erdogan F., 1981. Viscoelastic analysis of adhesively bonded joint. Journal of *Applied* Mechanics 48, 331–338.

Goland, M. and Reissner, E., 1944. The Stresses in Cemented Joints, Journal of Applied Mechanics 66, A17-A27.

Zakian, V., 1970. Optimization of numerical inversion of Laplace transform. Electronic Letters 6, 677-679.

Wang, J., and Zhang, C., 2010. A three-parameter elastic foundation model for interface stresses in curved beams externally strengthened by a thin FRP plate. *International Journal of Solids and Structures* 47, 998-1006.