COUPLED MASS AND ENERGY TRANSFER WITHIN TEXTRONIC STRUCTURES

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Abstract
Textronic structures are analyzed as textiles equipped with the textronic systems monitoring some medical parameters. The structures should be homogenized and the selected homogenization methods are discussed. The textronic systems can be implemented within the textiles. Both physical and mathematical models are determined for the different methods of textronic systems locations. Physical model is the homogenized textile structure with the holes of the defined shape or the textronic elements. Mathematical model consists of the state equations and the set of boundary and initial conditions. The problem can be solved analytically or by approximate methods as well as visualized by means of any processing program, cf. ADINA.
Key words: textronic, heat and mass transfer, modeling

1. Problem definition

Let us introduce the textronic structure, defined as the complex, composite textile structure of the clothing with the additional textronic elements. The monitoring textronic systems can be (i) located in the special channels within the textile material; (ii) introduced as the special conductive finish layer within the textile composite; (iii) introduced as the textronic elements of the higher density into the textiles. The main goal of the textronic system is to transmit the selected medical parameters from the human body to the analyzing unit. It should secure the thermal comfort of the user i.e. the optimal physical parameters within the different surrounding conditions. The measures of the thermal comfort are the state variables describing the current physical conditions within the clothing, i.e. the temperature as well as the water vapor concentration. The clothing can be also the personal protective equipment during the work. The main purpose is additionally to secure the adequate security level connected with the current parameters of the user's organism.

2. Physical model

Textiles are the products of the compact, inhomogeneous structure obtained by the different techniques, cf. the fabrics of different weaves, the knitted fabrics of different morfologies and the non-wovens. Textile composites consist of the textile products, the additional elements (the membranes, the waterproof layers, the special layers, etc.) as well as the air within the yarns and the free spaces between yarns. Thus, we homogenize the structure to create the physical model. The obtained homogeneous structure has the same conditions of the mass and heat transfer within the whole domain. The substitute heat and mass transfer coefficients are obtained by using the same methods, we can discuss only the heat transfer problem.

Thermal conductivity of the porous-capillary-passage materials was discussed by Wawszczak [1]. The substitute coefficient is the function of textile material λ_{mat}, as well as the gas within the free spaces between the material λ_{wyp}. Author define for this structure the coefficients in the direction parallel λ_{rw} and perpendicular λ_{rw} to the passages according to the formula

$$\lambda_{\mathrm{rw}} / \lambda_{\mathrm{mat}} = 1 - \xi\left(1 - \lambda_{\mathrm{wyp}} / \lambda_{\mathrm{mat}}\right) \qquad ; \qquad \lambda_{\mathrm{pr}} / \lambda_{\mathrm{mat}} = \frac{1}{1 - \xi + \dfrac{\xi}{\lambda_{\mathrm{wyp}} / \lambda_{\mathrm{mat}}}} \tag{1}$$

where ξ is the porosity factor defined as the volumetric ratio between the free spaces and the textile material. The non-systematic, mixed structures are defined according the same author by the substitute coefficient in the following form

$$\lambda_z = \delta_1 \lambda_{rw} + \delta_2 \lambda_{pr} \quad ; \quad \delta_1 + \delta_2 = 1; \tag{2}$$

where δ_1 and δ_2 are the coefficients dependent on the material according the added diagrams.
Tomeczek [2] discussed the composite of the fibres made of the reinforced material situated regularly within the filling. We assume that the heat transfer coefficient of the fibres is λ_{mat}, whereas the filling λ_{wyp}. The fibres have the regular cylinder shape of the radius r, which is in fact its volume fraction. The substitute coefficient is equal according to [2]

$$\lambda_z = \lambda_{wyp}\left(1 - r^{\frac{2}{3}}\right) + \frac{\lambda_{mat}\lambda_{wyp}}{2\lambda_{mat}\left(1 - r^{\frac{1}{3}}\right) + \lambda_{wyp} r^{\frac{1}{3}}} \; r^{\frac{2}{3}}. \tag{3}$$

Golański, Terada i Kikuchi [3] has introduced two methods to determine the heat transfer coefficient. The first is the classic *rule of mixture* of the substitute coefficient

$$\lambda_z = \lambda_{mat}\, \xi_{mat} + \lambda_{wyp}\, \xi_{wyp} \; ; \; \xi_{mat} = \frac{V_{mat}}{V_{mat} + V_{wyp}} \; ; \xi_{wyp} = \frac{V_{wyp}}{V_{mat} + V_{wyp}}, \tag{4}$$

where ξ_{mat} and ξ_{wyp} are the coefficients for the textile material of the volume V_{mat} and the free spaces of the volume V_{wyp}. The same authors proposed the second model – *Turner's model* – developed by means of the hydrostatic analogy. The substitute coefficient is now equal to

$$\lambda_z = \frac{\lambda_{mat}\, \xi_{mat}\, K_{mat} + \lambda_{wyp}\, \xi_{wyp}\, K_{wyp}}{\xi_{mat}\, K_{mat} + \xi_{wyp}\, K_{wyp}}. \tag{5}$$

where K_{mat} is the volumetric strain modulus of the textile material, K_{wyp} is the volumetric strain modulus of the filling.
The choice of scale introduces the next simplifications. The textile composite in the macro-scale is the complex structure and the coupled heat and mass transfer is defined as the 3D problem. The great number of net elements describing the shape of clothing causes the time-consuming calculations. The problem can be solved by means of approximate solutions.

2. Mathematical model

The most efficient description can be determined by means of Ostrogradski-Gauss theorem for the continuous integrands. State variables are now the temperature T, the water vapor concentration within the fibres w_f and the water vapor concentration within the free spaces between fibres w_a. Both mass and heat energy balances are the following correlations

$$\begin{cases} (1-\varepsilon)\dfrac{dw_f}{dt} + \varepsilon\dfrac{dw_a}{dt} = -\mathrm{div}\mathbf{q}_w + f_w; \qquad \mathbf{q}_w = D\,\nabla w_f + \mathbf{q}_w^*. \\ c\dfrac{dT}{dt} + \lambda_w \dfrac{dw_f}{dt} = -\mathrm{div}\mathbf{q} + f; \qquad \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^*. \end{cases} \tag{6}$$

where f_w is the mass source capacity, ε is the effective porosity of the material, $\boldsymbol{q}_w$ is the vector of mass flux density, $D = h_a \varepsilon/\zeta$ denotes the mass transport coefficient within the fibers, h_a is the coefficient of the water vapor transport within the fibers, ζ is the effective tortuousness of fabric, $\boldsymbol{q}_w^*$ is the vector of the initial mass flux density, f is the heat source capacity, c is the volumetric heat capacity of fabric, λ_w is the cross transport coefficient describing the sorption of water vapor by the fibers, $\boldsymbol{q}$ is the vector of heat flux density, $\boldsymbol{A}$ denotes the matrix of thermal conduction coefficients, $\boldsymbol{q}_w^*$ is the vector of the initial heat flux density.
Thus, the third equation is necessary to determine the state variables. We introduce the two-stage physical model of the sorption process according Li, Luo [4] and Li [5]. The first stage is the simple Fickian diffusion described as follows

$$\frac{dw_f}{dt} = (1-p)R_1 + pR_2 \qquad (7)$$

where p is the factor of the proportionality (i.e. the weight factor) in the form

$$\begin{aligned} &p=0 \quad \text{for } w_a<0{,}185 \text{ and } t<t_{eq}; \\ &p=1/2 \quad \text{for } w_a \geq 0 \text{ and } t<t_{eq}; \\ &p=1 \text{ for the optional } w_a \text{ and } t>t_{eq}; \end{aligned} \qquad (8)$$

and the equilibrium time t_{eq} is the function of the textile material, cf. the wool t_{eq}=540s see Li and Luo [4]. The sorption rate and the mass diffusion within the radial direction of the fibres is described by Crank [6] for the first stage as follows

$$R_1(\mathbf{x},t) = \frac{dw_f}{dt} = \frac{1}{r}\frac{d}{dr}\frac{(r\, D_f\, dw_f)}{dr}. \qquad (9)$$

Mass concentration is the same on the fibers surface as well as within the free spaces between fibers. Thus, the vapor concentration within fibers is the solution of Eq.(9) in the form

$$w_f(\mathbf{x}, R_1, t) = \rho\, w_a; \qquad (10)$$

Coupled heat and mass transfer within textronic structures during the first stage of the sorption process can be described by the Eqs.(6) and Eq.(10). Introducing the time derivative of Eq.(10) $\frac{dw_f}{dt} = \rho \frac{dw_a}{dt}$, we obtain the following correlations

$$\begin{cases} \eta\left(1-\varepsilon+\dfrac{\varepsilon}{\rho}\right)\dfrac{dw_f}{dt} = -\mathrm{div}\mathbf{q}_w + f_w; \quad \mathbf{q}_w = D\,\nabla w_f + \mathbf{q}_w^*; \\ c\dfrac{dT}{dt} + \lambda_w \dfrac{dw_f}{dt} = -\mathrm{div}\mathbf{q} + f; \quad \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^*. \end{cases} \qquad (11)$$

The second stage of the sorption process can be formulated by the experimental correlation

$$R_2(\mathbf{x},t) = s_1\, \mathrm{sign}(H_a - H_f)\exp\left(\frac{s_2}{|H_a - H_f|}\right); \qquad (12)$$

where s_1; s_2 are the experimental coefficients which depend on the mass of textile structure, cf. for example Li, Luo [4]. Of course, the sorption rate has the complicated mathematical form. Let us simplify the description of the mass transport between the material and the free spaces between fibres. We define according Li [5] the same saturated vapor pressure within the void spaces and on the fiber surface E_a=E_f as well as the same temperature T_a=T_f. Thus, we obtain the same proportional factor which is in fact the absorption coefficient η

$$\frac{w_f}{w_a} = \frac{\vartheta \frac{e_f}{T_f}}{\vartheta \frac{e_a}{T_a}} = \frac{e_f}{e_a} = \eta; \quad \frac{H_f}{H_a} = \frac{\frac{e_f}{E_f} \cdot 100\%}{\frac{e_a}{E_a} \cdot 100\%} = \frac{e_f}{e_a} = \eta. \tag{13}$$

where *w* is the absolute humidity, *e* is the water vapor pressure, *T* denotes the temperature, *H* is the relative humidity, υ is the number. Let us introduce Eq.(13) into Eq.(12). The obtained correlation is too complicated to solve it analytically and should be simplified. Let us differentiate Eq.(13) with respect to time. We obtain the following state equations

$$\begin{cases} \eta\left(1-\varepsilon+\dfrac{\varepsilon}{\eta}\right)\dfrac{dw_f}{dt} = -\mathrm{div}\mathbf{q}_w + f_w; \quad \mathbf{q}_w = D\,\nabla w_f + \mathbf{q}_w^*; \\ c\dfrac{dT}{dt} + \lambda_w \dfrac{dw_f}{dt} = -\mathrm{div}\mathbf{q} + f; \quad \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^*. \end{cases} \tag{14}$$

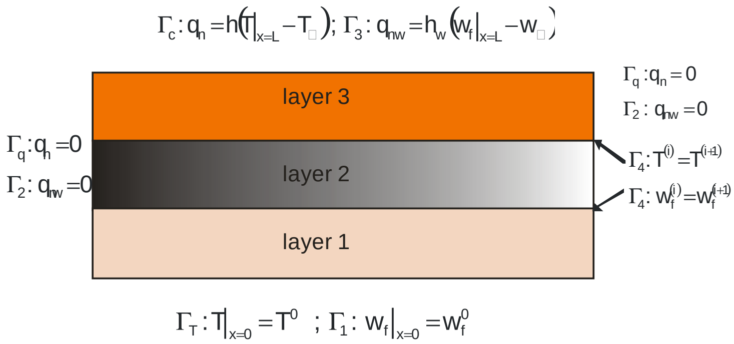

Figure 1. Boundary conditions of coupled heat and mass transfer

The state equations for the first stage Eq.(11) and the second stage of the sorption Eq.(14) are very similar. To solve the problem we should introduce the set of boundary conditions (i.e. first-, second- and third-kind conditions on the external boundary portions and fourth-kind conditions on the adjacent layers (i) and (i+1), see Fig.1), as well as the initial conditions

$$\begin{gathered} T(\mathbf{x},t) = T^0(\mathbf{x},t) \;\; \mathbf{x}\in\Gamma_T; \quad w_f(\mathbf{x},t) = w_f^0(\mathbf{x},t) \;\; \mathbf{x}\in\Gamma_1; \\ q_n(\mathbf{x},t) = q_n^0(\mathbf{x},t) \;\; \mathbf{x}\in\Gamma_q; \quad q_{nw}(\mathbf{x},t) = q_{nw}^0(\mathbf{x},t) \;\; \mathbf{x}\in\Gamma_2; \\ q_n(\mathbf{x},t) = h[T(\mathbf{x},t) - T_\infty(\mathbf{x},t)]\,\mathbf{x}\in\Gamma_C; \; q_{nw}(\mathbf{x},t) = h_w[w_f(\mathbf{x},t) - w_{f\infty}(\mathbf{x},t)]\,\mathbf{x}\in\Gamma_3; \\ T^{(i)}(\mathbf{x},t) = T^{(i+1)}(\mathbf{x},t) \;\; \mathbf{x}\in\Gamma_4; \quad w_f^{(i)}(\mathbf{x},t) = w_f^{(i+1)}(\mathbf{x},t) \;\; \mathbf{x}\in\Gamma_4; \\ T(\mathbf{x},0) = T_0(\mathbf{x},0) \;\; \mathbf{x}\in(\Omega\cup\Gamma); \quad w_f(\mathbf{x},0) = w_{f0}(\mathbf{x},0) \;\; \mathbf{x}\in(\Omega\cup\Gamma). \end{gathered} \tag{15}$$

The notations are the following: T^0 and w_f^0 are the state variables within the thin air layer between the skin and the textile composite, q_n and q_{nw} denote the heat and mass flux densities normal to the boundary, h and h_w are the heat and mass convection coefficients on the external boundary of the composite, T_∞ and $w_{f\infty}$ are the state variables in the surrounding, T_0 and w_{f0} are the initial state variables at the beginning of the process.

4. Solution of coupled heat and mass transfer

The solution of the coupled problem is simplified for the steady heat and mass transfer. The state variables are constant in time and the state equations can be formulated as follows

$$\begin{cases} -\operatorname{div}\mathbf{q}_w + f_w = 0; & \mathbf{q}_w = D\,\nabla w_f + \mathbf{q}_w^*; \\ -\operatorname{div}\mathbf{q} + f = 0; & \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^*. \end{cases} \tag{16}$$

Let us for simplicity assume the vanishing heat and mass sources within the real structure $f=0$, $f_w=0$; as well as the vectors of the initial mass flux density $\boldsymbol{q}_w^*=0$ and initial heat flux density $\boldsymbol{q}_{nw}^*=0$. Let us next introduce that the textile composite is made of the one layer. The design variable is now the thickness of the structure described by the coordinate *x*. State equations (16) can be reformulated to the following

$$\frac{d^2T}{dx^2} = 0; \quad \frac{d^2w_f}{dx^2} = 0. \tag{17}$$

The solution can be obtained by the simple integration with respect to *x* as follows

$$T_{,xx}=0 \;;\; T_{,x}=C_1 \;;\; T=C_1x+C_2;$$
$$w_{f,xx}=0 \;;\; w_{f,x}=C_1 \;;\; w_f=C_1x+C_2. \tag{18}$$

The integration constants are determined by means of mixed boundary conditions. There are the first-kind conditions on the boundary near the skin (x=0), and the third-kind conditions on the boundary contacting the surrounding (x=L). The conditions can be described as follows

$$T\big|_{x=0} = T^0 \quad ; \quad w_f\big|_{x=0} = w_f^0$$
$$q_n = \lambda\varepsilon \frac{dT}{dx}\bigg|_{x=L} = h\left(T\big|_{x=L} - T_\infty\right) \;;\; q_{nw} = D\varepsilon \frac{dw_f}{dx}\bigg|_{x=L} = h_w\left(w_f\big|_{x=L} - w_{f\infty}\right). \tag{19}$$

Introducing the conditions Eqs.(19) within the state equations (18), we obtain the functions

$$w_f = \frac{h_w\left(w_f^0 - w_{f\infty}\right)}{\frac{h_a\varepsilon}{\zeta}\varepsilon - h_w L}x + w_f^0; \quad T = \frac{h\left(T^0 - T_\infty\right)}{\mathbf{A}\varepsilon - hL}x + T^0 \tag{20}$$

We see at once that the state variables are the linear functions of the coordinate *x*. The state variables on the external boundary of the structure for $x=L$ are greater than within the air layer between the skin and composite for $x=0$.

The more complicated structures should be solved approximately, by means of different numerical methods. The state equations are defined by Eqs.(16) and the boundary conditions Eqs.(15). The solution strategy is shown in Fig.2. Let us introduce the special conductive finish layer within the textile composite. Thus, we homogenize the typical sandwich structure made of three layers of the finite element net shown in Fig.3.

The layer#1 is characterized by: isotropic thermal conductivity of fibers $\mathbf{A}^{(1)} = 0{,}15\,\mathrm{W/(mK)}$; heat sorption of water vapor by fibers $\lambda_w^{(1)}$=3550 J/kg; volumetric heat capacity of dry fibers $c^{(1)}=1610\cdot10^3$J/(m^{3}K), effective porosity ε=0,925; effective tortuousness of fabric ζ=1,20; density of fibers ρ=1320kg/m^3. The parameters of the layer#2 are: isotropic thermal conductivity of fibers $\mathbf{A}^{(2)} = 0{,}10\,\mathrm{W/(mK)}$; heat sorption of water vapor by fibers $\lambda_w^{(2)}$=7000J/kg, volumetric heat capacity of dry fibers $c^{(2)}=3000\cdot10^3$J/(m^{3}K), effective porosity ε=0,925; effective tortuousness of fabric ζ=1,50; density of fibers ρ=2000kg/m^3. The parameters of the layer#3: isotropic thermal conductivity of fibers $\mathbf{A}^{(3)} = 0{,}21\,\mathrm{W/(mK)}$; heat sorption of water vapor by fibers $\lambda_w^{(3)}$=2530J/kg; volumetric heat capacity of dry fibers $c^{(3)}=1610\cdot10^3$J/(m^{3}K), effective porosity ε=0,925; effective tortuousness of fabric ζ=1,20;

density of fibers ρ=1320kg/m^3; mass convection coefficient h_w=0,15m/s; heat convection coefficient h=0,1W/(m^{2}K).

Problem definition

Physical model

Mathematical model

Introduction of (i) procedure solution of differential equation
(ii) boundary and initial conditions

Analysis stage: Finite Element Method
4-nodal, 3D elements

Synthesis stage: Finite Difference Method

Visualization of results = maps of
(i) temperature distribution, (ii) moisture distribution.

Figure 2. Solution strategy of the coupled heat and mass transport

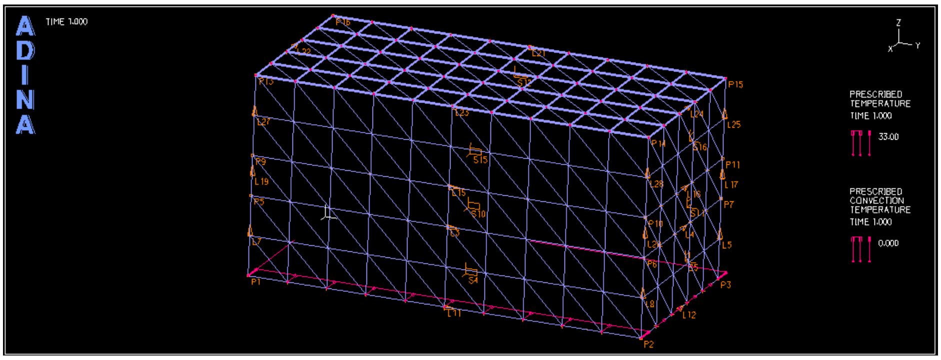

Figure 3. Space net of finite elements

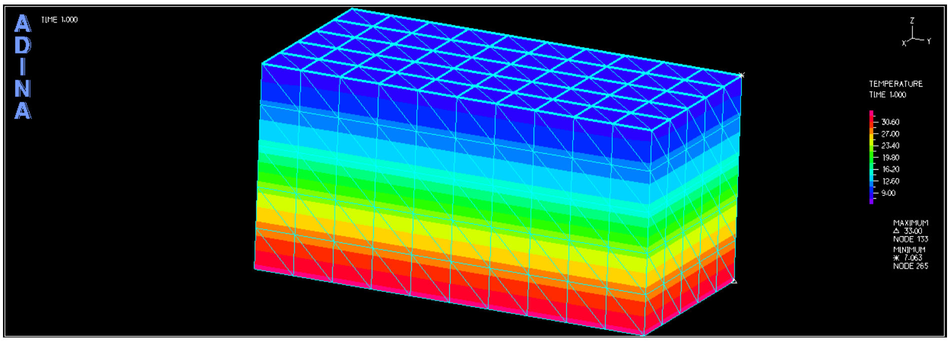

Figure 4. Distribution of temperature

The obtained state variables are visualized by the ADINA program which has the graphic modulus. The distributions of the temperature is shown in Fig.4 whereas the water vapor concentration (i.e. the moisture concentration) in Fig.5. We have to apply the space net because the program does not contain the linear function of the heat and the mass convection. We introduce also the upper surface subjected to the convection.

The obtained sandwich distribution has the small irregularities caused by the accuracy errors within the elements. The intermediate layer has the increased resistance against the permeability of heat and water vapor. There is also the model of membrane introduced into the textile structure. The obtained distributions shows that the intermediate layer has almost the same value of both state variables.

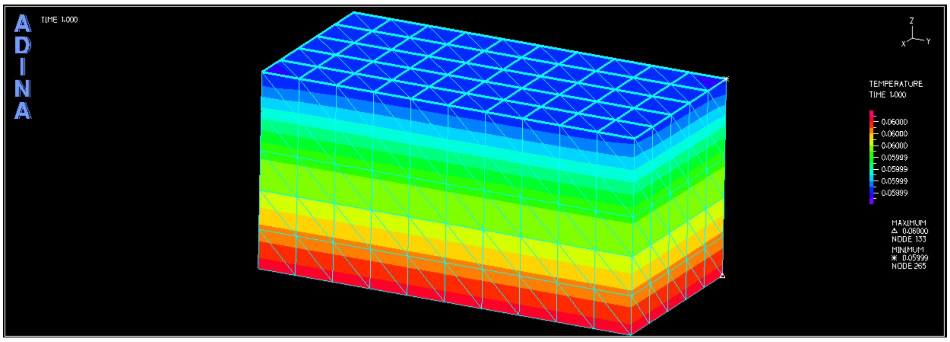

Figure 5. Distribution of water vapor concentration

Let us introduce the complex textile composite with the free spaces. The homogenized textile material is composed of the textiles and the air between the fibres. The external part of the structure was subjected to the finishing procedure and is waterproof of the thickness equal to 9% of the total thickness of the structure. The surface of the hole is coated by the plastic of the panchromatic emissivity $\varepsilon=0,9$. The temperature of the free space between the skin and the composite $T_1=32^oC=305K$; the surrounding $T_\infty=-3^oC=270K$.

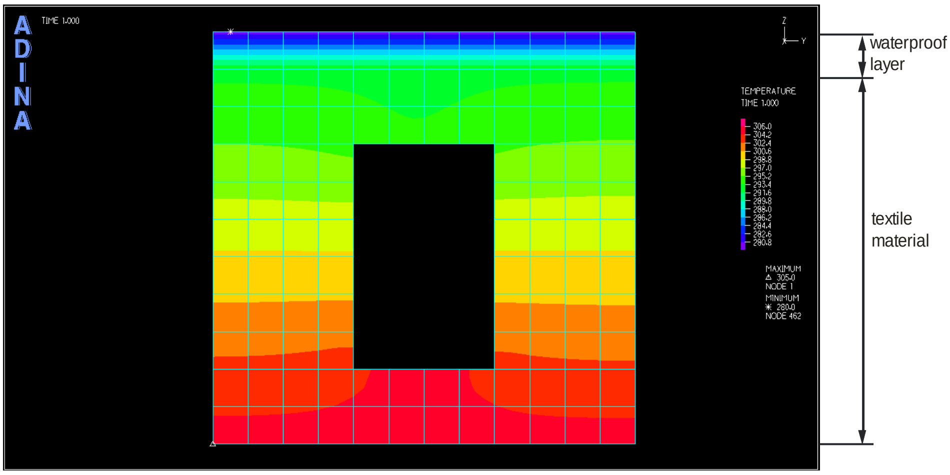

Figure 6a. Distribution of temperature for t=20s

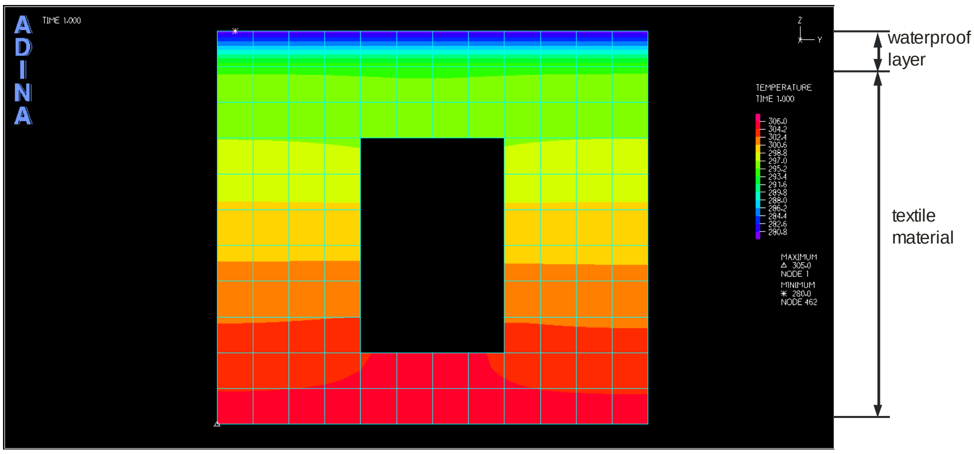

Figure 6b. Distribution of temperature for t=40s

The material is isotropic of the thermal conductivity of fibers ***Λ****=0,055 W/(mK)* and the same parameter for the waterproof layer is ***Λ****=0,09 W/(mK)*. The heat convection coefficient is acc. Zarzycki [7] $h=99,4\ W/(m^2K)$. The heat capacity of the dry fibres is $c=1609,7\cdot 10^3\ J/(m^3K)$. The structure was discretized by means of the finite element net made of 9-nodal elements.
The temperature field for the times *t=20s* and *t=40s* are shown in Fig.6. The temperature has the sandwich distribution. The analogical distribution has the water vapor concentration within the structure and is not shown in this paper.

5. Conclusions

Coupled heat and mass transport is the main factor of the clothing comfort as well as the safety of the user during the extreme environmental conditions. Normal wear conditions can be described by means of the steady problems whereas the dynamic problems by the transient description. Steady problems are relatively simple and some of them can be solved analytically. The transient problems are the most complicated and should be solved approximately by means of the different numerical methods.
State equations are the heat energy and mass balances as well as the additional experimental equation. Problem should be supplemented by the set of boundary and initial conditions. The analysis of the physical phenomena within the composite textile structure and on the external and internal boundaries allows to formulate all the equations. Distribution of state variables can be visualized by means of any graphical program, for example ADINA.
The presented methods are an effective tool to generate the distribution of temperature and water vapor concentration in textile structures. Distribution of state variables is in fact the analysis stage of the problem. The synthesis stage allows to optimize or identify the textile structure subjected to the coupled heat energy and mass transport, by implementation of the various optimization methods.

6. Acknowledgements

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