THEORETICAL EXPRESSION OF COMPRESSIBILITY BEHAVIOUR OF WARP KNITTED SPACER FABRICS

Fatemeh MOKHTARI[1], Mahnaz SHAMSHIRSAZ[2]
Masoud LATIFI[1], Mohammad Maroufi[2]
[1] *Textile Engineering Department, Textile Research and Excellence Centers, Amirkabir University of Technology, Tehran, Iran*
2 New Technology Research Center, Amirkabir University of Technology, Tehran, Iran
E-mail:matt@aut.ac.ir

Abstract
Nowadays, 3-D spacer fabrics characteristics become a highly interested concept for textile researchers. These products have extensive application in automobile, locomotive, aerospace, building and other industries. Several techniques could be applied to produce spacer fabrics using woven, weft and warp knitting technology. Warp knitting is the most commonly used technology for production of spacer fabrics. Spacer fabrics present special characteristics compared to conventional textiles due to their wonderful porous 3-D structures. The compression resistance is a distinct feature beneficial for the structural stability of spacer fabrics, and it is proper to fulfil permanent or instant loading and recovery requirements.
The goal of this research is to develop an theoretical model to predict the compressibility behaviour of warp knitted spacer fabric and compare it with experimental data. All required samples were produced on two needle bars Rachel warp knitting machine with different thickness, stitch densities and texture designs. The basic theory is based on modelling pile yarns as curve bar. The theoretical results related to the assumption of pile yarn as curve bar are closer to the experimental results of these spacer fabrics comparing to the previous theoretical models in literature.
Key words: Spacer fabrics, Compressibility behaviour, Curved bar, Theoretical model

Introduction

Compressibility is known as an important mechanical property of textiles. Compressibility is a decrease of initial thickness that occurs with suitable increase of compressive force. Initial thickness of tissue is considered as the thickness without applied force on the touch screen. In Murthyguru theory, compressibility refers to a reduce of fabric thickness with a required pressure increase on fabric, the textiles thickness depends on the applied pressure at measuring time [1]. Textiles thickness depends on the applied pressure at measuring time. The most important and fundamental theory related to the compressibility of textiles is Van Wyk theory. As we spend a lot of time sitting, not to mention roughly a third of our lives in bed, this is a good reason to place a high value on the construction and components of bed and seat cushioning which play an important role for a comfortable and healthy life. The cushioning should match the shape of the human body and individual sleeping or seating promote rest and relaxation during the day or night, and help to create a feeling of well-being throughout the day. This is a job for good, all-round cushion materials, which can meet the complex profile of comfort requirements relating to their mechanical deformation behaviour to their mechanical deformation and climatic physiology [2].
The relatively high compression resistance offered by spacer fabrics ensures stability of the product in the third direction (perpendicular to its surface), preventing deformation of the outer layers. spacer yarns are presented as highly oriented in the total structure .it should be noted here that the flexibility of conventional “single layer” fabrics, comprises their main properties, making them particularly suitable for the “external” repair of damage concrete members[3,4].
Many works have been done on modelling of compression behaviour of warp knitted spacer fabrics, with different structural shape assumption and then presenting resulted stress-strain curve[2,3,4]. But the initial curvature and deformation of monofilament during loading have not been considered yet. Therefore, the goal of this research is to provide a theoretical expression which could predict the compression behaviour of warp knitted spacer fabrics.

Theoretical Models

Model (a): Deflection of the vertical bars in compression

In this case, assuming the monofilaments as the vertical bars as presented in fig.1, applying a distributed pressure load on fabrics, the changes in the connecting bar height occur [5].

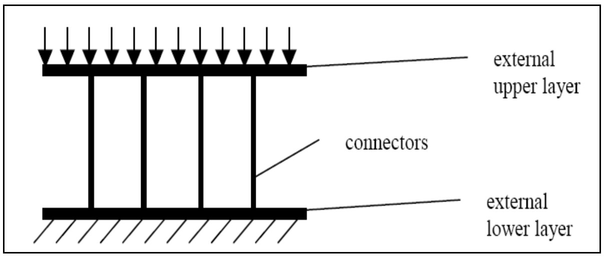

Figure 1.Monofilaments as the vertical bars in compression [5]

The critical load for buckling of vertical bars is determined by

$$P_{cr} = \frac{\pi^2 EI}{l^2} \quad (1)$$

Where E is Modulus, I is the moment of inertia and L is Length.

Model (b): Deflection of the curved bars in compression

In this case, the monofilaments have the initial curved shapes before loading fig.2. The theory is based on the small deformations of an elastic coplanar curved bar presented in the literature [6]. Any point on a typical cross section of such a bar is displaced from its unstrained position through small components u ,v and w along x, y and z directions (fig. 3). u ,v and w are the continuous single-value functions of the coordinates and, in the case of a curved bar,u and v represent tangential and radial displacements of points on the cross section and w represent the displacement in a direction normal to the plane of the bar (the z direction). The plane of cross section remain plane after deformation and that the bar material is linearly elastic, homogeneous, isotropic, and continuous. This allows to take advantage of the principle of superposition and to study each of these displacement components separately.

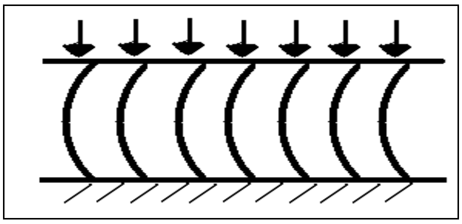

Figure 2. Monofilaments as the curved bars in compression

The absolute displacement of a point is, of course, the vector sum of the three independent components. In fact, the complete deformation of a bar element can be considered to be due to three types of displacement :u_0 ,a displacement of points on the cross section due to a stretching of the centroidal axis;u_1, a shortening of the fibres due to a decrease in the radius of curvature R by an amount v ; and u_2, a normal displacement due to rotations of the cross-sectional plane about the y and z axes. These displacements are illustrated in fig. 3.

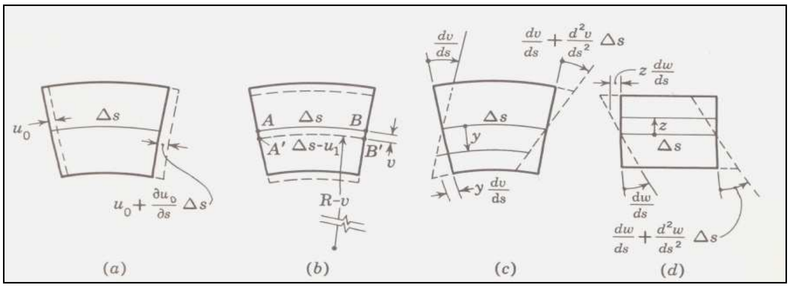

Figure 3. Deformation of a curved bar element[6]

For bending moments, we have

$$M_y = -E\left[J_{YZ}\left(\frac{d^2v}{ds^2}+\frac{v}{R^2}\right)+J_y\frac{d^2w}{ds^2}\right] \tag{2}$$

$$M_z = -E\left[J_z\left(\frac{d^2v}{ds^2}+\frac{v}{R^2}\right)+J_{yz}\frac{d^2w}{ds^2}\right] \tag{3}$$

Note that owing to the lack of symmetry of the cross section, a change in curvature in the xy plane produces bending moments in the xz plane and vice versa.

EJ_z

M_y and M_z to change in curvature .the quantities EJ_y, and EJ_{yz} are called elemental flexural stiffness, or more often, flexural rigidities of the bar, and EA is called the elemental axial stiffness(or the axial rigidity) of the bar. The reciprocal of the elemental stiffness are called elemental flexibilities.

Finally, according to equation (2),(3) and the reference equations, the alternative form of the differential equation of the elastic curve have been obtained.

$$-E\frac{d^2}{ds^2}\left[J_{yz}\left(\frac{d^2v}{ds^2}+\frac{v}{R^2}\right)+J_y\frac{d^2w}{ds^2}\right]+P_z=0 \tag{4}$$

$$-E\left(\frac{d^3}{ds^3}+\frac{R'}{R}\frac{d^2}{ds^2}+\frac{1}{R^2}\frac{d}{ds}\right)\left[J_z\left(\frac{d^2v}{ds^2}+\frac{v}{R^2}\right)+J_{yz}\frac{d^2w}{ds^2}\right]+\frac{dP_y}{ds}+\frac{R'}{R}P_y=0 \tag{5}$$

Where $R'=\frac{dR}{ds}$ and v is the displacement radial direction.

For the model developed in this paper, it is assumed that R is constant i.e. the bar is circular, so the terms containing R, obviously vanish in the above equations [5]. The general solution of earlier differential is given as

$$v = c_1\cos\varphi + c_2\sin\varphi + c_3\varphi\cos\varphi + c_4\varphi\sin\varphi + c_5 \tag{6}$$

Any of the above differential equations must satisfy a number of independent static and/or kinematic boundary conditions equal to the sum of the orders of the equations. Five boundary conditions are needed to specify the unique solutions. Assuming that the element is symmetric, we have:

(i) $$v\Big|_{\varphi=-\alpha} = 0$$

So equation (6) change to

$$c_1 cos\alpha - c_2 sin\alpha - c_3 \alpha cos\alpha + c_4 \alpha sin\alpha + c_5 = 0 \quad (7)$$

As the other boundary conditions, we have

(ii) $$M\Big|_{\varphi=\alpha} = 0$$

(iii) $$M\Big|_{\varphi=\alpha} = 0$$

The equivalent force is:

(iv) $$P = N cos\alpha - V sin\alpha$$

The shear force is:

$$V_y = \frac{dM_z}{ds} = \frac{dM}{Rd\varphi} = \frac{1}{R}\frac{dM}{d\varphi} \quad (8)$$

$$N = -\frac{dV_y}{d\varphi} \quad (9)$$

So for (i) and (ii) we obtain:

$$M = -EJ_z\left[\frac{d^2v}{ds^2} + \frac{v}{R^2}\right] = -\frac{EJ_z}{R^2}\left[\frac{d^2v}{d\varphi^2} + v\right] \quad (10)$$

Therefore, we have

$$\frac{d^2v}{d\varphi^2} + v\Big|_{\varphi=\alpha} = 0 \quad (11)$$

$$\frac{d^2v}{d\varphi^2} + v\Big|_{\varphi=-\alpha} = 0 \quad (12)$$

And the force P can be determined by

$$P = -\frac{dV_y}{d\varphi} cos\alpha - \frac{1}{R}\frac{dM}{d\varphi} sin\alpha \quad (13)$$

$$P = -\frac{dV_y}{d\varphi} cos\alpha - \frac{1}{R} sin\alpha * \frac{EJ_z}{R^2}\left[\frac{d^3v}{d\varphi^3} + \frac{dv}{d\varphi}\right] \quad (14)$$

Also, assuming that the curve beam is symmetrical, so at the center of the beam, as the last boundary condition, we have

$$\text{(v)} \quad \left.\frac{dv}{ds}\right|_{\varphi=0} = 0$$

So the form of equation (6) would be changed to

$$-c_1 sin\varphi + c_2 cos\varphi + c_3 cos\varphi - c_3\varphi sin\varphi + c_4 sin\varphi + c_4\varphi cos\varphi = 0 \quad (15)$$

So, by the above five boundary conditions (i),(ii), (iii), (iv) and (v), the constants c_1, c_2,c_3,c_4 and c_5 could be calculated by above equetions[6].

Materials and Tests

The 3-D spacer fabric configuration studied in this paper is shown in figures 4 and 5. Two bi-directional woven face sheets are connected with vertical woven piles. According to the directions of the yarns, three types of warps and wefts have been considered for the studied structure. Two types consist of the straight warps and wefts individually in the X and Y directions of the top and bottom face sheet. The third type is the monofilaments which shuttle between the two face sheets in the Z direction to shape an array of c shape in warp and weft direction.

The monofilaments in the face sheets are in the warp direction, along which the 3-D spacer fabric is rolled up. The fabric structure design parameters can be selected and fabricated variably: the pile height, the distribution density of piles, the anisotropy distribution of the yarns in the warp and the weft directions.

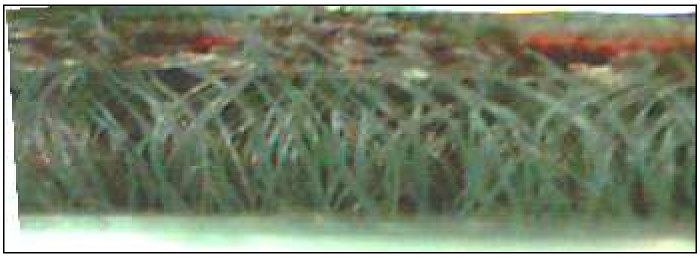

Figure 4.Spacer fabric in weft direction

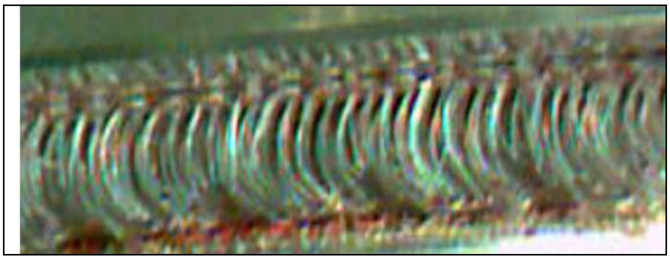

Figure 5. Spacer fabric in warp direction

In this work, the 3-D spacer fabric is woven with Polyester fiber, produced by Rachel Warp knitting machine with two needle bars with specific thicknesses, densities and texture designs. Characteristics of the this spacer fabric components are given in table 1.

Table 1. Characteristics of the spacer fabric samples

Thickness (mm)	Score Cotton (Tex)	Yarn type	density (cpc')	Type layer	sample
3.75	2.23 (monofilament)	Polyester	14	connectors	Raw fabric (3PT)
	11.22 (48 f)	Polyester		External upper layer	
	7.78 (34 f)	Polyester		External lower layer	

For sample, five specimens are selected for compression tests. The compression tests are carried out on Instron 5566. During loading, photos in weft and warp directions for different loading pressure are taken.

Results and Discussion

The stress-strain curve obtained experimentally has been illustrated in Fig. 6. The experimental results are demonstrated and compared with theoretical data calculated by model (a) in fig. 7. As the theoretical models are given for elastic deformations, it would be mentioned that in fig. 7 and fig. 8, only the elastic region of experimental stress-strain curve (region I in fig. 6) is shown and compared with theoretical results.

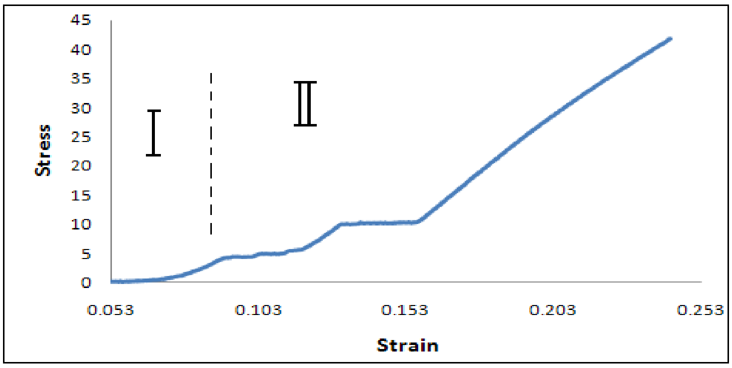

Figure 6. stress-strain curve obtained experimentallly for spacer fabrics in compression

Using model (a) with the assumption of vertical bars as monofilaments, there exists a significant deviation of theoretical results from experimental data, as it was expected. As it can be seen in fig. 4 and fig. 5, the monofilaments in the spacer fabric have the initial curved shapes and this curvature increases with increase of loading pressure. Despite of model (a), in model (b), not only the initial curvatures of monofilaments have been considered but also this curvature depends on applied force and increases with loading increase.

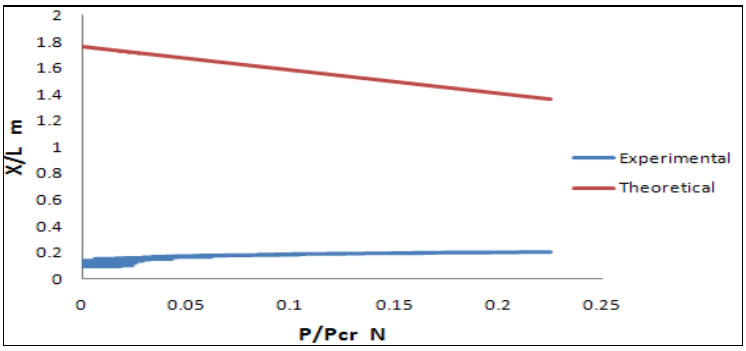

Figure 7. Comparison of experimental and theoretical results according to vertical beam theory (Model a)

In fig. 8, the theoretical results by model (b) are compared with experimental results of elastic region. However there exists always a significant difference between theoretical and experimental results, but it can be observed an improvement of model to predict the mechanical behaviour of these spacer fabrics in compression. This deviation shows that the proposed model (b) predicts a more flexible mechanical behaviour comparing to real mechanical behaviour of these fabrics in compression. This can be explained by (i) monofilaments have different shapes in weft and warp directions.(ii) the effect of two layers on both sides of pile yarns have been neglected.(iii) the proposed model is based on the bending of curved bars in the elastic region, so to have a global behaviour of these spacer fabrics, another model related for plastic deformation should be integrated in theoretical base model.

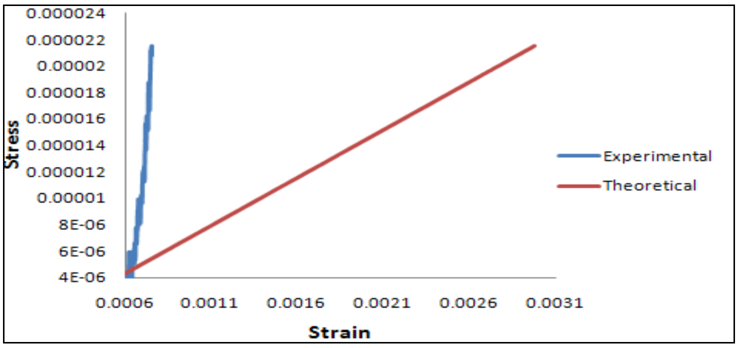

Figure 8. Comparison of experimental and theoretical results according to curved beam theory (Model b)

Conclusion

A theoretical model to predict the compressibility behaviour of spacer fabrics has been developed and proposed. The monofilaments of the spacer fabric studied in this paper have the initial curved shape in weft and warp directions. The proposed theoretical model considers the initial curvature and the variation of this curvature during compression of spacer fabric. The results show that the proposed model could better predict the compressibility behaviour of these fabrics in elastic region comparing to previous models in literature. Further investigations are going on way by the authors to achieve a model for the global compression behaviour (elastic- plastic) and the complexity of structural shape of the 3-D spacer fabrics.

Refrences

1. Murthyguru, *"Novel approach to Study Compression Properties in Textile"*, Autex Res. J., Vol. 5, No 4, 2005
2. M.Xu-hong,G.Ming-qiao.,*"The Compression Behaviour of the Warp Knitted Spacer Fabric*" Fiber and textile in Eastern Europe,Vol.16,No.1,pp-56-61, March 2008
3. S.Vassiliadis,A.Kallivretaki,N.Provatidis.,*"Numerical modeling of the compressional behavior of warp-knitted spacer fabrics"* Fiber and textile in Eastern Europe,Vol.17,No.5,pp-56-61,2009
4. Ming Li,Shaokai Wang,Zuoguang Zhang,Boming Wu*"Effect of Structure on the Mechanical Behaviors of Three-Dimensional Spacer Fabrics Composites"* Appel Compos Mater.Springer 2009.
5. M.Musiol, *"Analysis of Transverse Deformation Of Spacer Products"* AUTEX Research Journal,Vol.5.No1,March 2005
6. J.T.Oden.,*"Mechanics of Elastic Structures*",McGraw-Hill,inc,Usa,1967,pp-130.