MODELING OF TWO-PLY YARN STRAIN AT BREAK

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Abstract
The phenomenon of two-ply yarn strain at break is not too examined. If you want to mathematically describe two-ply yarn strain at break, you most likely find the most usual used formula, i.e. equivalence between single and two ply yarn strain at break. But in reality they have to differ, basically because of mechanism in single yarns during two-ply yarn twisting. They can be similar, but just in case of very low two-ply yarn twists.

The presented paper will try to introduce several mathematical models of two-ply yarn strain at break describing and compare them with experimental results. The models will come out of geometrical assumptions of two-ply yarn, in one case the possible two-ply yarn contraction will be counted with.

Key words: two-ply yarn, strain at break, geometrical model, polypropylene two-ply yarn

1. Introduction

The questions of two-ply yarn strain at break are not solved too frequently. The presented paper will introduce imagination about two-ply yarn geometry; behaviour of single yarn in two-ply yarn during twisting (plying) and consequently overview of existing models and new approaches will be appended.

2. Background Research

The literature [1] offers two basic relations for two-ply yarn strain at break description. The first one, let's mark it like M01, eq. (1), doesn't count with any changes of single yarn in two-ply yarn

$$\varepsilon_s \approx \varepsilon_j \,, \qquad (1)$$

where ε_s [-]… strain at break of ply yarn
ε_j [-]… strain at break of single yarn.

This is simple equivalence between these two types of linear textile products in point of view of strain at break parameter. The second equation, eq. (2) – M02 already contains influence of two-ply yarn parameters – two-ply yarn twist and two-ply yarn diameter (both through the angle β_s)

$$\varepsilon_s = \frac{\varepsilon_j}{\cos^2 \beta_s} \,, \qquad (2)$$

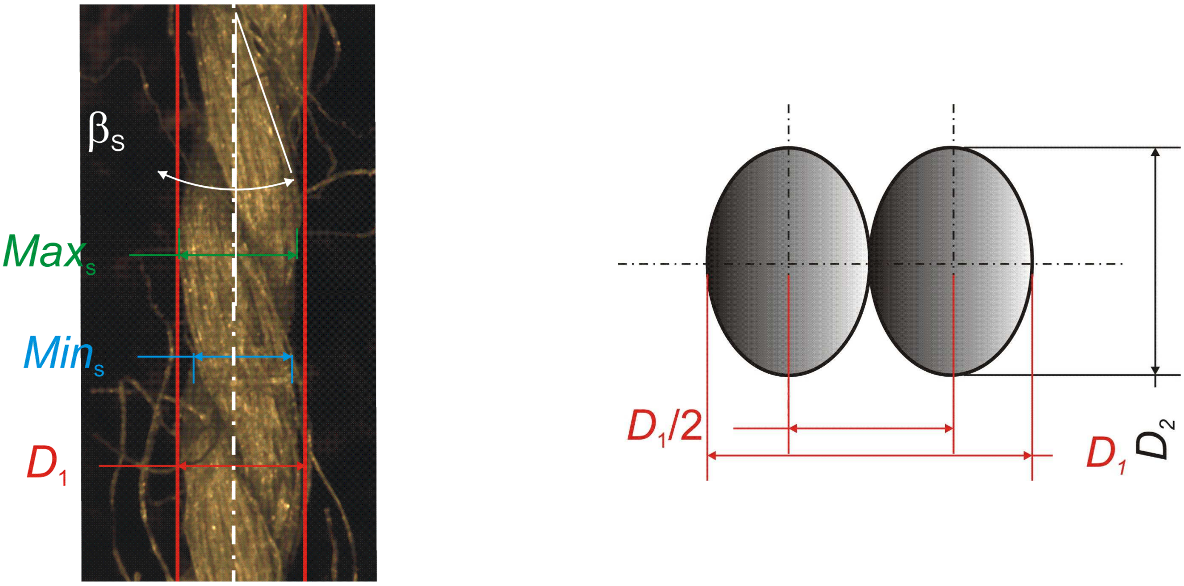
where β_s [rad]… angle between single yarn axis and ply yarn axis (see cap. 3, fig. 1), then $\tan\beta_s = \pi Z_s D_s / 2$,
Z_s [1/m]… ply yarn twist
D_s [m]… ply yarn diameter.

3. Geometry of Two-ply Yarn

Ply yarn (here concretely two-ply yarn) belongs to a group of linear textile formations, i.e. one proportion is higher than two remaining, but nevertheless it is 3D formation.

Shape of single yarn is simplified to cylinder, then the diameter of its bottom can be easily measured by known methods [2, 3, 4] for the diameter of single yarn.

By plying of two or more single yarns body with thin and thick parts originates (fig. 1a)), what two-ply yarn retreats from cylindrical shape.



a) b)

Figure 1. Show of two-ply yarn in a) projection, b) cross-section

On fig. 1a) three transversal proportions are visible, which is possible to measure, model eventually. This is parameter Min_S – the shortest perpendicular distance of ply yarn body bound in one row of image, Max_S – the longest perpendicular distance of ply yarn body bound in one row of image, D_1 (or D_S) – diameter of the smallest cylinder, where can be ply yarn placed. The most important parameter for further usage in case of projection is parameter D_1, D_S respectively, which can describe "diameter" of ply yarn.

4. Model of Two-ply Yarn Strain at Break – without Diameter Contraction

This purely geometric model assumes:

1. two-ply yarn is approximated by two helixes,
2. two-ply yarn is constructed from two single yarns with the same properties (mechanical and also geometrical),
3. then is possible to observe behaviour of just ¼ of two-ply yarn basic cell (see fig. 2),

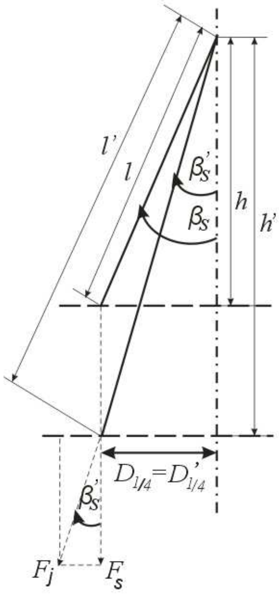


Figure 2. Simple model of ¼ of two-ply yarn without diameter contraction [5], with help of [6]

4. two-ply yarn twist has opposite direction to single yarn twist,
5. the diameter of two-ply yarn D_s is defined in cap. 3, distance of single yarn axes is $D_s/2$ and half of it then $D_s/4$ (here $D_{1/4}$),
6. at the moment of two-ply yarn break the contraction of single yarn diameters is not considered, so the two-ply yarn diameter D_s is not contracted also $D_{1/4} = D'_{1/4}$,
7. at the moment of two-ply yarn break the change of two ply yarn length is from h to h', of single yarn in two-ply yarn is from l to l' and angle between single yarn axis to two-ply yarn axis from β_s to β'_s,
8. at the moment of two-ply yarn break the two-ply yarn is loaded by force F_s and in single yarn acts force F_j.

On a basement of these assumptions and also fig. 2 is possible to derive following equations for two-ply yarn strain at break.
According to fig. 2 is valid:

$$\cos\beta_s = \frac{h}{l},$$

$$\cos\beta'_s = \frac{h'}{l'},$$

$$\varepsilon_s = \frac{h'-h}{h} \Rightarrow h' = h(1+\varepsilon_s),$$

$$\varepsilon_j = \frac{l'-l}{l} \Rightarrow l' = l(1+\varepsilon_j),$$

then

$$\cos\beta'_s = \frac{h'}{l'} = \frac{h(1+\varepsilon_s)}{l(1+\varepsilon_j)} = \cos\beta_s \frac{(1+\varepsilon_s)}{(1+\varepsilon_j)}.$$

If according to Pythagorean Theorem is valid

$$(D_{1/4})^2 = l^2 - h^2 \text{, then}$$

$$(D'_{1/4})^2 = l'^2 - h'^2 .$$

If we assume any diameter contraction, the diameters are equal before and after loading. So the same is valid also for their square.

$$l^2 - h^2 = l'^2 - h'^2 \Rightarrow 1 - \frac{h^2}{l^2} = \frac{l^2 (1+\varepsilon_j)^2}{l^2} - \frac{h^2 (1+\varepsilon_s)^2}{l^2},$$

$$1 - \cos^2 \beta_s = (1+\varepsilon_j)^2 - \cos^2 \beta_s (1+\varepsilon_s)^2,$$

$$\varepsilon_s = \sqrt{1 + \frac{2\varepsilon_j + \varepsilon_j^2}{\cos^2 \beta_s}} - 1. \tag{3}$$

If we consider small deformation of two-ply yarn,

$$2\varepsilon_s + \varepsilon_s^2 \cong 2\varepsilon_s,$$

then it is possible to simplify eq. (3)

$$\varepsilon_j = \sqrt{1 + 2\cos^2 \beta_s \left(2\varepsilon_s + \varepsilon_s^2\right)} - 1 \cong \sqrt{1 + 2\varepsilon_s \cos^2 \beta_s} \cong \left(1 + \varepsilon_s \cos^2 \beta_s\right) - 1 = \varepsilon_s \cos^2 \beta_s,$$

$$\varepsilon_s = \frac{\varepsilon_j}{\cos^2 \beta_s}. \tag{4}$$

So this last equation (4) corresponds to eq. (2) in cap. 2.

5. Model of Two-ply Yarn Strain at Break – with Diameter Contraction

The assumptions for this model are practically the same like by previous model, except points 6 and 7 (see cap. 4). The diameter of two-ply yarn D_s (its ¼ respectively) at the moment of its break is considered to change from $D_{1/4}$ to $D''_{1/4}$, length of single yarn changes from l to l'' and length of two-ply yarn from h to h'', β_s to β''_s. Comprehensive scheme to this situation is visible in fig. 3.

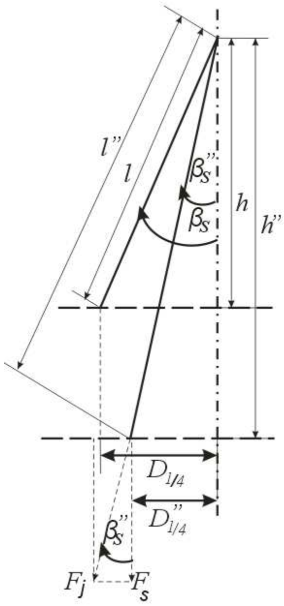


Figure 3. Simple model of ¼ of two-ply yarn with diameter contraction [5]

The appropriate equations to this situation follow.

$D_{1/4} > D''_{1/4}$, we can use coefficient k>1, $D_{1/4}\,k = D''_{1/4}$.

$$\cos\beta_s = \frac{h}{l},$$

$$\cos\beta''_s = \frac{h''}{l''},$$

$$\varepsilon_s = \frac{h''-h}{h} \Rightarrow h'' = h(1+\varepsilon_s),$$

$$\varepsilon_j = \frac{l''-l}{l} \Rightarrow l'' = l(1+\varepsilon_j),$$

$$\left(D''_{1/4}\right)^2 = k^2\,(l''^2 - h''^2),$$

$$l^2 - h^2 = k^2\left[l^2\,(1+\varepsilon_j)^2 - h^2\,(1+\varepsilon_s)^2\right],$$

$$1-\cos^{2\beta_s} = k^2\left[(1+\varepsilon_j)^2 - \cos^2\beta_s\,(1+\varepsilon_s)^2\right],$$

$$\varepsilon_s = \frac{1}{k\cos\beta_s}\sqrt{k^2\,(1+\varepsilon_j)^2 - \sin^2\beta_s} - 1. \qquad (5)$$

In the next part of article the experimental values will be compared with these models and by the last one the optimal value of *k* will be searching.

6. Experimental Part

6.1 Overview of Material for Experiment

Previous models, see tab. 1, will be compared with experimental values of polypropylene two-ply yarns, see tab. 2.

Table 1. Summary of models

Description	Model Number	Equation
Equivalence between single and two-ply yarn strain at break	M01	$\varepsilon_s \approx \varepsilon_j$ (1)
Simplify Model without any Diameter Contraction	M02	$\varepsilon_s = \frac{\varepsilon_j}{\cos^2\beta_s}$ (2)
Model of Two-ply Yarn Strain at Break without Diameter Contraction	M03	$\varepsilon_s = \sqrt{1+\frac{2\varepsilon_j+\varepsilon_j^2}{\cos^2\beta_s}} - 1$ (3)
Model of Two-ply Yarn Strain at Break with Diameter Contraction	M04	$\varepsilon_s = \frac{1}{k\cos\beta_s}\sqrt{k^2\,(1+\varepsilon_j)^2 - \sin^2\beta_s} - 1,\quad k > 1$ (5)

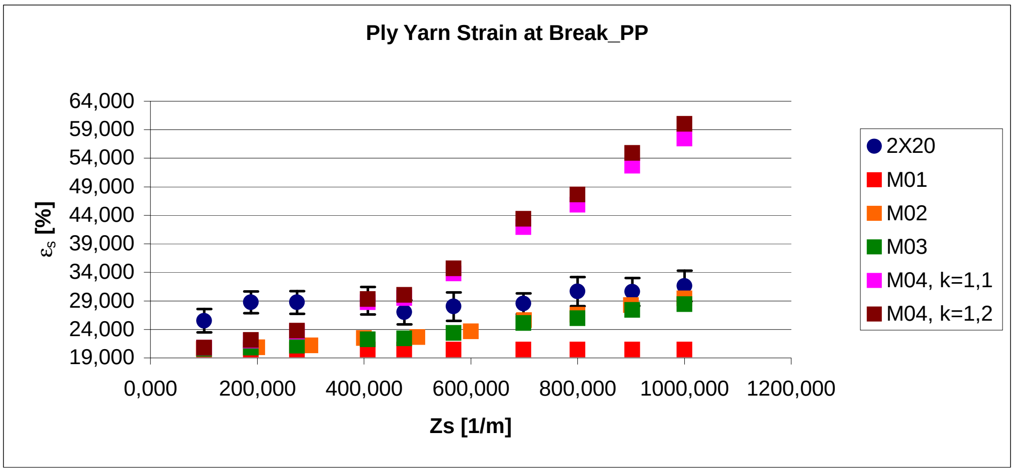
Table 2. Overview of experimental two-ply polypropylene yarns

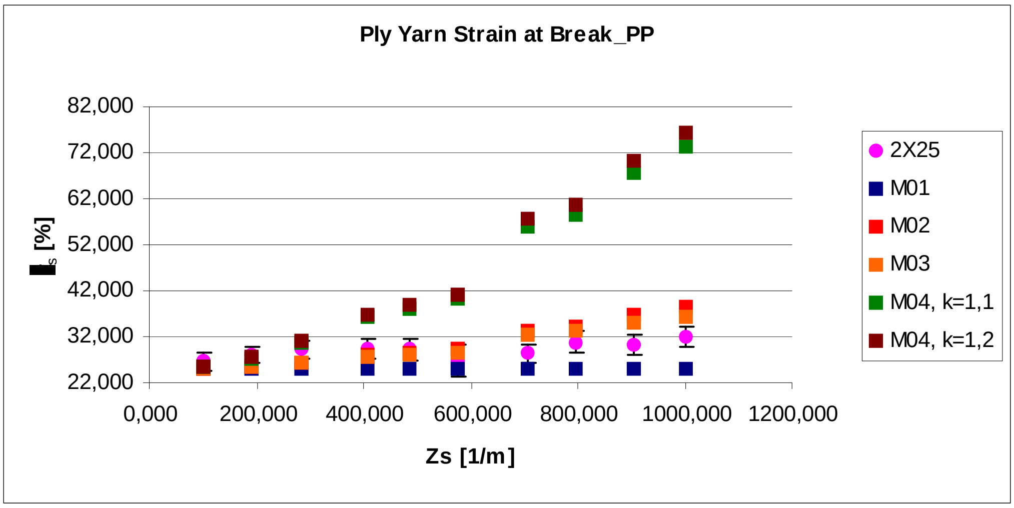
Single Yarn, 100% PP	T_j [tex]/ Z_j [1/m]	T_s [tex]	Z_s [1/m]									
2,2 [dtex]/50 [mm]	20/720	2x20	100	200	300	400	500	600	700	800	900	1000
	25/620	2x25										
	29,5/560	2x29,5										
	35,5/475	2x35,5										
	45/410	2x45										

Where T_j is single yarn fineness, Z_j is single yarn twist T_s two-ply yarn fineness and Z_s two-ply yarn twist (all named parameters are nominal). Greyed two-ply yarns in tab. 2 are normally produced by company, rest of them where made in laboratory conditions. Crossed cells indicate, that these yarns weren't possible to produce.

6.2 Comparison of Experimental and Model Values

Comparison is realized graphically in fig. 4.





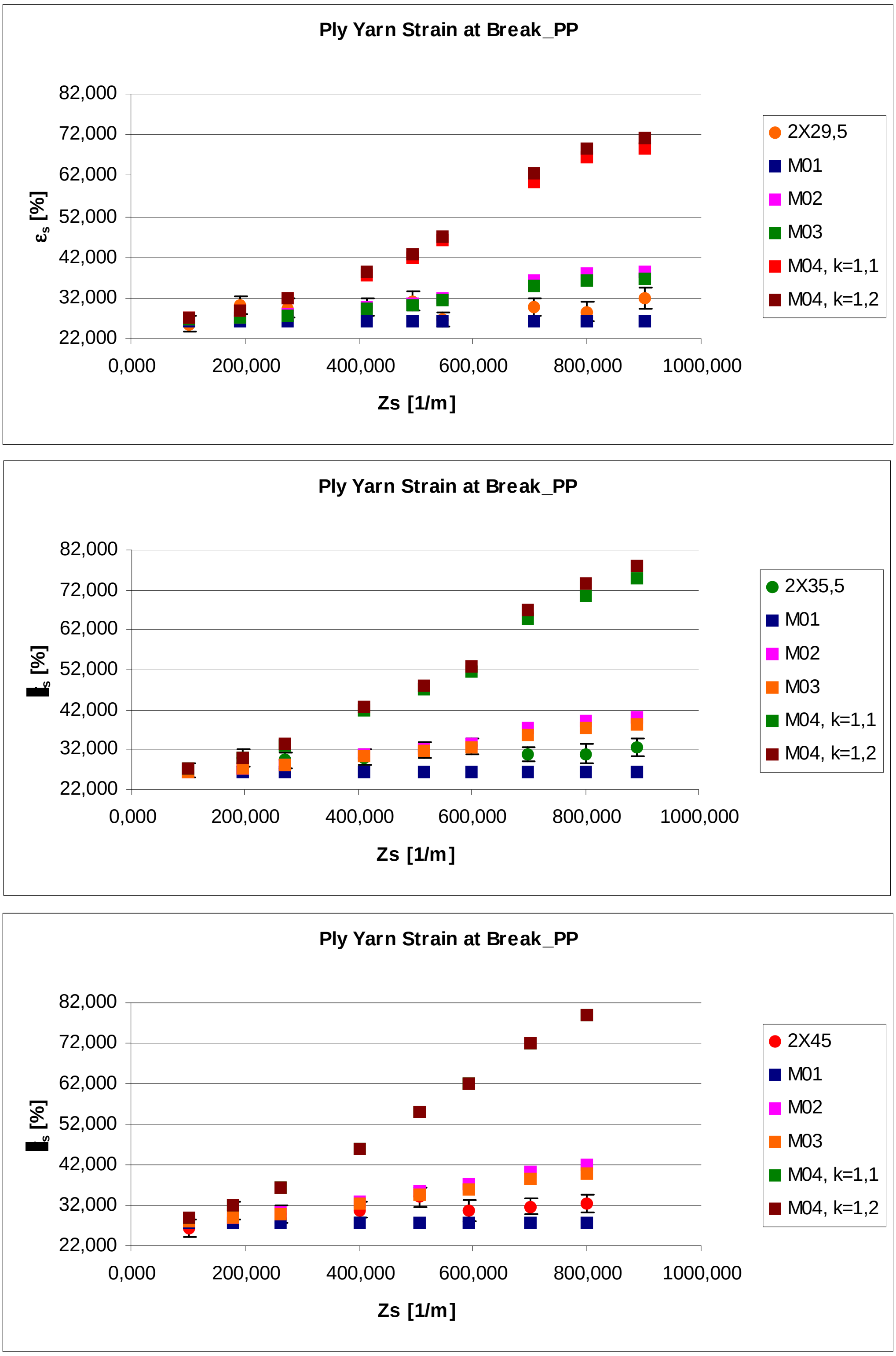


Figure 4. Graphical comparison of models and experiment

7. Discussions and Conclusion

The experimental results show the same increasing trend with ascendant ply yarn twists for all tested two-ply yarns. So this fact eliminates the first model M01, see tab. 1, three remaining models are still in play. Unfortunately even assumption about diameter contraction at two-ply yarn break wasn't most likely correct, see graphs in fig. 4. The eq. (4) – M04, the calculated values according to this relation respectively, doesn't fit the experimental values. Its progress in dependence on ply yarn twists is far faster than by experimental values, it is visible already by the smallest parameter $k = 1.1$.

The last two models M02, M03 consider the influence of ply yarn twist and omit diameter contraction. Both are suitable for experimental values, but the model M03 (which talks about two-ply yarn strain at break like big deformation) is significantly closer to experimental values than the model M02.

It is necessary to note, that the comparison of experimental and model values was realized in common two-ply yarn twists range, i.e. 200 – 600 1/m. The rest of yarns was produced in laboratory conditions just for information about behaviour out of the standard boundary.

8. References

1. Ursíny, P.: Cotton Spinning Technology II., VŠST, Liberec, 1991 (lecture notes in Czech)
2. Apparatus Zelweger Uster Tester, http://www.uster.com/UI/textile-Yarn-Testing-2-183.aspx, cited 05. 08. 2010
3. Křemenáková, D., Rubnerová, J.: Internal Standard 22-103-01/01 – Yarn Packing Density Direct and Secant Method, Liberec, 2001
4. Voborová, J., Neckář, B.: Internal Standard 22-108-01/01 – Yarn Diameter, Liberec, 2001
5. Mihulova, R.: Modeling of Breaking Strain of Two-ply Polypropylene Yarn, Diploma Thesis, TUL, Liberec, 2009
6. Neckar, B.: Morfologie a strukturni mechanika obecnych vlakennych utvaru, TUL scriptum, Liberec, 1998