Comparative Study on Modeling of Cellular Materials under Large Deformation

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Introduction

Cellular materials are widely being used in structural as well as non-structural applications in aerospace applications. Typical cellular materials possess plasticity like stress plateau while undergoing large compressive deformation. Because of this characteristic, they are suitable for cushioning or buffering applications. The stress strain relationship under different impact velocity conditions is to be modeled accurately in order to input in FE codes, to simulate different impact safety and crashworthiness simulations. The choice of the most suitable density for the selected type of foam is based on stress–strain behavior, obtained by means of experimental tests and/or models. [3]

Different models has been proposed and published in literature for accurate modeling of mechanical response of cellular materials [1 to 3], the objective of each model being the same, but they differ in terms of number of parameters, accuracy and flexibility. A comparative study on selective available material models is carried out in order to model the response of the cellular materials under large deformation.

Models

Most of the models used for numerical simulations are phenomenological models and have simple parameter identification based on fitting of experimental data [3]. Three models are being compared in the present study namely Schraad, Liu and Avalle Models.

A model by Schraad and Harlow [1], in which the finite compressive strain is related to stress by a finite strain Young's modulus, $E(\varepsilon)$. $E(\varepsilon)$ is itself dependent on the parent material stiffness E_s and relative density $\phi(\varepsilon)$ of the foam, and a shape factor $A(\varepsilon)$, and is given by:

$$E(\varepsilon) = A(\varepsilon) E_s [\phi(\varepsilon)]^2$$

Where

$$\phi(\varepsilon) = \frac{\phi_o}{1+\varepsilon}$$

$$A(\varepsilon) = A_o H(\varepsilon - \varepsilon_y) + A_1(\varepsilon_y - \varepsilon) + (A_2 - A_1) H(\varepsilon_d - \varepsilon)$$

Where ϕ_o is the initial solid volume fraction of cellular material. ε_y and ε_d are the yield and densification strains respectively. While elastic-plateau and plateau-densification transition occurs through a range of strains ($2\Delta\varepsilon_y$ & $2\Delta\varepsilon_d$, respectively.)

Liu and Subhash [2] have proposed a simple phenomenological constitutive model that is capable of capturing the tensile and compressive response of low density foams. It is given as follows

$$\sigma = A\frac{e^{\alpha\varepsilon}+1}{B+e^{\beta\varepsilon}} + e^C(e^{\gamma\varepsilon}+1)$$

The parameter A is indicative of compressive yield stress, while α and β control the hardening/softening behavior of the stress– strain curve. B controls the onset of plasticity under tension and the expression $e^C(e^{\gamma\varepsilon}+1)$ defines the rapid increase in stress during the densification stages of compressive deformation.

The Avalle Model [3] is also a phenomenological model and has been developed in order to better fit the experimental stress–strain curve.

$$.\,\sigma = A(1-e^{(-E/A}) \varepsilon (1-\varepsilon)^m + B\left(\frac{\varepsilon}{1-\varepsilon}\right)^n$$

A and *E* define the yield stress and elastic modulus respectively. The second addendum is introduced in order to have a vertical asymptote corresponding to the physical limit of compression (ε =1).

Experimental Tests

Static uni-axial compression tests were performed on closed cell Aluminum foam specimens, dia 25mm and height of 25mm, of three different densities (200, 300 & 400 kg/m^3). A hydraulic universal testing machine was used; with a strain rate of 2x10^{-3}/s.

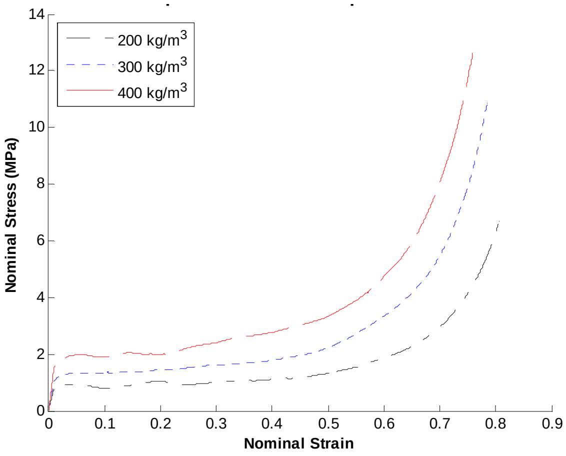

Fig. 1. Experimental stress-strain curves.

Comparison

Parameters

Table 1. Model Parameters

Model	Total
Schraad	$7: A_o, \varepsilon_y^{\uparrow}, \Delta\varepsilon_y, A_1, \varepsilon_d^{\uparrow}, \Delta\varepsilon_d, A_2$
Liu	$6: A^{\uparrow}, B^{*}, \alpha, \beta, C, \gamma$
Avalle	$5: A^{\uparrow}, E^{\uparrow}, m, B, n$

* B=1 as proposed by Liu *et el.* [2].

↑ Initial guess is input through experimental curve.

Prediction

The scheme proposed by Liu *et el.* [2,5] is used to determine the model parameters through a nonlinear fitting function, called *nlinfit* in Matlab®.

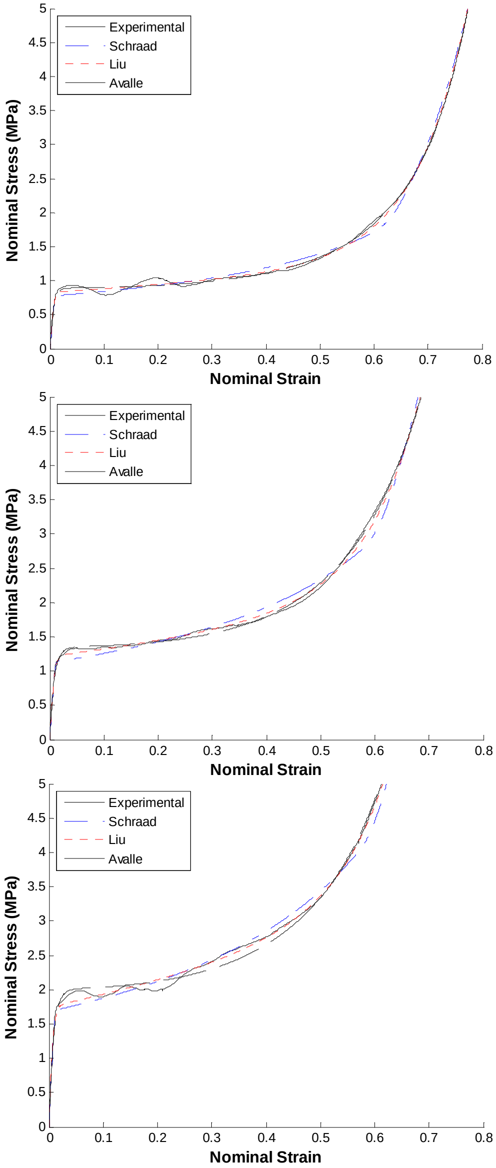

Fig. 2. Predictive curves using different models, (a) Schraad Model (b) Liu Model (c) Avalle Model

A quantitative comparison is carried out on the basis of mean squared error (MSE), where

$$MSE = \frac{1}{n}\sum_{i}^{n}(\sigma_{i,predicted} - \sigma_{i,experimental})^2$$

Table 2. Calculated MSEs (%)

Model	200 kg/m^3	300kg/m^3	400 kg/m^3
Schraad	0.77	2.20	1.51
Liu	0.22	0.43	0.34
Avalle	0.16	0.14	0.81

Flexibility

Schraad and Avalle Model can predict only one type of load type i.e. compression or tension at a time. While Liu model can represent both simultaneously.

Discussion

The purpose of current study is not to declare any model as an accurate one, but is to propose a methodology to compare different available models for some specific application. The suitability of the model may depend on the requirement of the user e.g. foam type, load types to be predicted or number of parameters and maximum parameters which may be guessed through experimental curve to avoid higher MSE or extended convergence time.

Conclusion

It is clear that each model, compared in current work, is capable of modeling the compressive response for different densities of Aluminum foams with sufficient accuracy (MSE ≤ 2.2%). However, it may be concluded that a model having less number of parameters may give better output in terms of MSE provided its parameters are sufficient to predict each regime of stress strain curve i.e. elastic, plateau and densification. Hence in current study Avalle Model with total five parameters, out of which two are guessed through experimental curve, may be considered suitable.

References

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