LATERAL AND RESTRAINED DISTORTIONAL BUCKLING OF DOUBLY-SYMMETRIC STEEL I-BEAMS

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Introduction

Steel beams with thin-walled open sections are prone to changes in the cross-section geometry, known as distortions, prior to and during buckling. In intermediate-length I-beams with stocky flanges and slender web, lateral buckling of the beam may be accompanied by the distortions in the web, creating a coupled buckling mode, called lateral distortional or simply distortional buckling (DB). A special type of DB, known as restrained distortional buckling (RDB), takes place in beams with top or bottom flange having insufficient lateral restraint. For instance, a beam with an effective lateral brace that only supports the top flange at the loading point may undergo web distortions in the vicinity of the loading point which cause the bottom flange to undergo excessive lateral deflection while the top flange does not deform laterally.

The web distortions in DB cause reductions in the torsional and warping rigidities of steel I-beams, which cause reductions in the buckling moment. The present structural steel codes [1, 2, 3] do not consider these reductions.

The present study involves analytical and numerical studies for investigating the DB behavior of doubly-symmetric steel I-beams and the reductions in their buckling moments due to web distortions. DB moment equations were developed for doubly-symmetric steel I-beams based on the studies in the literature [4, 5, 6, 7]. These equations not only apply to both elastic and inelastic DB but also consider the reductions in the torsional and warping rigidities due to web distortions. The estimates from these solutions are compared to the numerical results obtained for simply-supported doubly-symmetric steel I-beams with a concentrated load and a lateral top brace at midspan.

Analytical Study

Two different DB solutions were proposed in the present study. According to the first solution, which was developed based on the LTB solution of the AS4100 code [3] and the findings of Bradford [6], the DB moment (M_d) of a doubly-symmetric steel I-beam is given by

$$M_d = 0.6M_p\left[\sqrt{\left(M_y/M_{ed}\right)^4+3}-\left(M_y/M_{ed}\right)^2\right]\le M_p \qquad (1)$$

where M_y and M_p are the yield and plastic moments of the section, respectively; and M_{ed} is the elastic DB moment, calculated from

$$M_{ed} = \frac{C\cdot\pi}{L}\cdot\sqrt{EI_y\cdot GJ_e+\left(\frac{\pi}{L}\right)^2\cdot EI_y\cdot EI_{we}} \qquad (2)$$

where L is the unbraced span length; C is a coefficient accounting for the loading and support conditions; EI_y is the lateral flexural rigidity; and GJ_e and EIw_e are the effective torsional and warping rigidities, respectively, accounting for the reductions in the rigidities due to web distortions. Pi and Trahair [4, 5] proposed the following equations for simply-supported beams in uniform bending:

$$GJ_e = \left(24\cdot GJ_f\cdot D_wL^2/\pi^2d\right)\Big/\left(\left(2GJ_f\right)+\left(12D_wL^2/\pi^2d\right)\right) \qquad (3)$$

where GJ_f is the torsional rigidity of the flange; d is the beam depth; and $D_w=E.t_w^3/[12.(1-\nu^2)]$ with E and ν being the elastic modulus and Poisson's ratio of steel, respectively; and t_w the web thickness.

$$EI_{we} = EI_w\Big/\left(1+r_{fw}^3\cdot\left(d/12L\right)\cdot\left(1+b_f/d\right)\right) \qquad (4)$$

where EI_w is the warping rigidity; b_f is the flange width; and r_{fw} is the smaller of t_f/t_w and 2 with t_f being the flange thickness. Eq. (1) is applicable to elastic and inelastic buckling and yielding limit states.

In the second solution, the DB moments of I-beams are calculated from two different equations in the elastic and inelastic ranges of buckling, as opposed to the first solution. Elastic DB moment (M_{ed}) is obtained from Eq. (2) while inelastic DB moment (M_{id}) is obtained from the following equation which is based on the inelastic LTB formula proposed by Nethercot and Trahair [7]:

$$M_{id} = M_p\left[0.7+\frac{0.3\cdot\left[1-0.7\cdot M_p/\left(\alpha_m\cdot M_{od}\right)\right]}{0.61-0.3\beta_m+0.07\beta_m^2}\right]\le M_p \qquad (5)$$

where β_m is the ratio of the end moments of the unbraced span; α_m is a moment modification factor accounting for the nonuniform moment distribution in the span and obtained from α_m=1.75+1.05β_m+0.3β^2_m<2.56; and M_{od} is the reference DB moment corresponding to uniform moment in the unbraced span.

Numerical Study

Previously, Zirakian and Showkati [8] were able to obtain both DB and RDB modes in the span of steel I-beams they tested. In the present study, steel I-beams with cross-sectional dimensions and material properties identical to Specimen S210-5200 of Zirakian and Showkati [8] and with varying unbraced lengths were analyzed using the finite element program ANSYS [9]. The beams were simply-supported in and out of plane at the ends and subjected to a central concentrated load. An effective lateral brace was provided at the top of the beams so that the beams underwent RDB at midspan and DB at quarter spans [8]. Hexagonal elements with a 10-mm edge length were used in the model and eigenvalue buckling analysis was performed to obtain the buckled shapes, buckling modes and buckling moment values numerically.

Comparison of the Analytical and Numerical Results

The numerical results obtained from the finite element analysis (FEA) were compared to the estimates obtained from the two DB solutions proposed in this study (Eq. 1 and Eqs. 2&5) and from the lateral torsional buckling (LTB) solutions of the AISC-LRFD [1], EC3 [2], and AS4100 [3] codes. The results are given in Fig. 1.

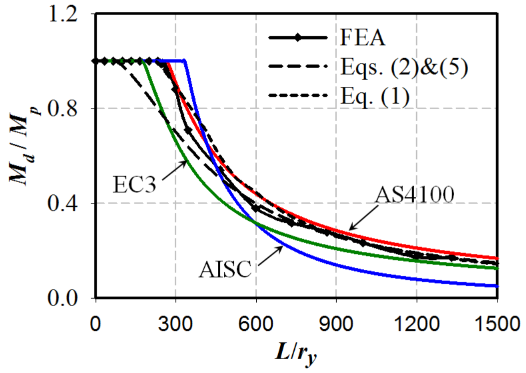

Fig. 1 Comparison of FEA results with the proposed DB solutions and the LTB solutions of the codes

Both of the DB solutions provided close estimates to FEA results, particularly in the elastic range of buckling. In the plastic range and the transition zone between the plastic failure and inelastic DB (L/r_y<500), the second solution (Eqs. 2&5) yielded to estimates in closer agreement with FEA results compared to the first solution (Eq. 1). For L/r_y values greater than 500, the first solution gave closer estimates to the FEA results.

The LTB solutions of the codes generally provided conservative estimates with the AISC solution being overly conservative in the elastic range of buckling. The EC3 solution provided conservative estimates in the entire range of structural response. Although being slightly on the unconservative side, the AS4100 solution was found to be in the closest agreement with the FEA results compared to the other code solutions. The analysis showed that the LTB solutions of the three codes provide buckling moment estimates, which are generally on the conservative side.

Finally, the relationship between the web slenderness (h/t_w) and the reduction in the buckling moment due to web distortions was investigated. The beams with the cross-sections, web slenderness, flange slenderness ($b_f/2t_f$), and lateral slenderness (L/r_y) values given in Table 1 were analyzed using the two proposed solutions. The lateral slenderness values of the beams were chosen such that the unbraced length of each beam was 1.5 times its limit length for elastic buckling given in the AISC-LRFD [1] code. It was found out that the reduction from the LTB moment (M_b) increased with increasing web slenderness linearly (Fig. 2).

Table 1 The analyzed beams

Section	$b_f/2t_f$	h/t_w	L/r_y
W44x335	4.5	38.0	401
W36x330	4.5	31.4	428
W36x210	4.5	39.1	397
W30x148	4.4	41.6	393
W21x166	4.6	25.0	478

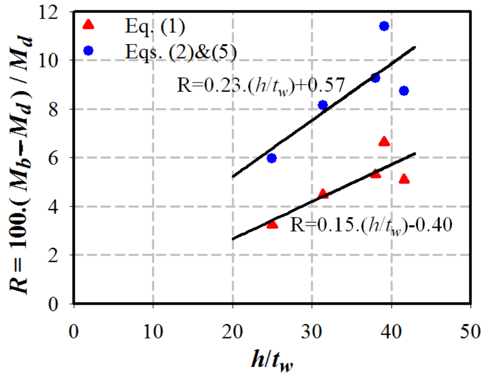

Fig. 2 Reductions in the buckling moments for various web slenderness values

Conclusions

The DB and RDB behavior of simply-supported doubly-symmetric steel I-beams with a central concentrated load and an effective lateral brace at midspan was investigated. The proposed solutions were found to be in closer agreement with FEA results compared to the LTB solutions of the structural steel codes. The reductions in the buckling moments of I-beams due to web distortions were shown to increase with increasing web slenderness for a constant flange slenderness.

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