# THERMOMAGNETOELASTIC BEHAVIOR OF A ROTATING FUNCTIONALLY GRADED HOLLOW CYLINDER

**Zengtao Chen, A. Hamid Akbarzadeh**

Department of Mechanical Engineering, University of New Brunswick, Fredericton, Canada, E3B 5A3

## Introduction

Recently, thermo-magneto-elastic interactions have attracted significant attention in many applications, such as smart structures, magnetoelastic sensors, magnetostrictive materials, optics, geophysics, and plasma physics. The interaction among thermal, electrical, magnetic, and elastic fields is usually encountered in various applications of piezomagnetic and piezoelectric materials [1]. On the other hand, functionally graded materials (FGMs) with continuously changing material properties have been developed for high temperature environment [2]. As a result, understanding the thermomagnetoelastic response of FGM structures is crucial for many industries.
The present work is mainly motivated by Babaei and Chen [3], who provided an exact solution for an infinitely long, magnetoelectroelastic, homogenous, hollow rotating cylinder. The aim of this paper is to provide a closed form solution for the thermomagnetoelastic behavior of a rotating FGM hollow cylinder through a straightforward method.

## Problem Description and Formulation

The cylinder is subjected to the temperature gradient, $\vartheta = T - T_\infty$, where $T$ is the absolute temperature at the inner and outer surfaces and $T_\infty$ is the ambient temperature; Furthermore the cylinder is exposed with the magnetic potential $\varphi$, as well as internal and external pressures. The constitutive equations for a linear thermomagnetoelastic medium are expressed as:

$$\sigma_{ij} = C_{ijkl}\bar{\varepsilon}_{kl} - d_{kij}H_k - \beta_{ij}\vartheta$$
$$B_i = d_{ikl}\bar{\varepsilon}_{kl} + \mu_{ik}H_k + \tau_i\vartheta$$

where $\sigma_{ij}$, $B_i$, $\bar{\varepsilon}_{ij}$, and $H_k$ are, respectively, the stress, magnetic induction, strain, and magnetic field; $C_{ijkl}$, $d_{kij}$, $\mu_{ik}$, $\beta_{ij}$, and $\tau_i$ are the elastic, piezomagnetic, , and magnetic permeability coefficients, thermal moduli, and pyromagnetic coefficients, respectively. The linear strain-displacement relations and the quasi-stationary magnetic field equations in the absence of free conducting currents are:

$$\bar{\varepsilon}_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),\ H_i = -\varphi_{,i}$$

in which, $u_i$ and $\varphi$ are, respectively, the components of displacement and magnetic scalar potential; a comma represents partial differentiation with respect to the space variable. The cylinder is assumed to be cylindrically orthotropic and infinitely long. The non-zero components of stress and magnetic induction for the axisymmetric, plane strain problem are written as:

$$\sigma_{rr} = c_{33}u_{,r} + c_{13}\frac{u}{r} + d_{33}\varphi_{,r} - \beta_1\vartheta,$$
$$\sigma_{\theta\theta} = c_{13}u_{,r} + c_{11}\frac{u}{r} + d_{31}\varphi_{,r} - \beta_3\vartheta$$
$$B_r = d_{33}u_{,r} + d_{31}\frac{u}{r} - \mu_{33}\varphi_{,r} + \tau_1\vartheta$$

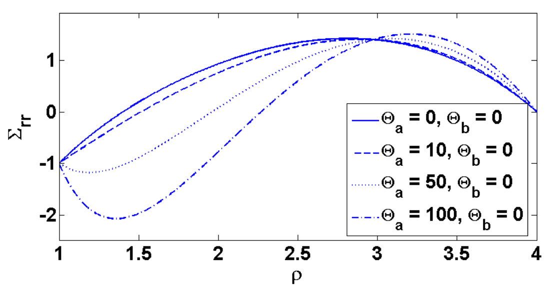
where $u = u_r$ is the radial displacement; $r$ and $\theta$ are the radial and circumferential coordinates; $c_{mn} = C_{ijkl}$, $d_{mk} = d_{pij}$ ($i, j, k, l, p$ = 1, 2, 3; $m, n$ = 1, 2,…, 6), $\beta_1 = \beta_{11}$, and $\beta_3 = \beta_{33}$. The governing equations for the rotating cylinder are:

$$\sigma_{rr,r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho_d u_{,tt}$$
$$B_{r,r} + \frac{1}{r}B_r = 0$$

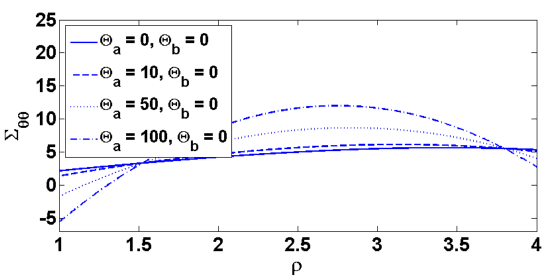
where $\rho_d$ is the mass density, and t stands for time. In addition, the material properties are graded according to a power law along the radial direction. Temperature distribution is obtained by solving the steady-state heat conduction equation in the radial direction. Two fully coupled second-order governing differential equations in terms of displacement and magnetic potential are solved directly by a straightforward decoupling method.

## Results and Discussion

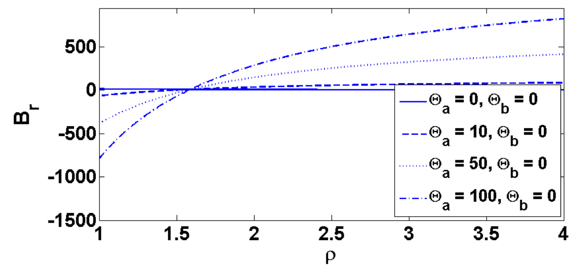
Numerical results are displayed to illustrate the effects of temperature gradient, angular velocity, non-homogeneity parameter, and aspect ratio on the thermomagnetoelastic response. The results include radial and hoop stresses, magnetic potential, magnetic induction, and temperature gradient in a non-dimensional form. As an example, an FGM hollow cylinder with outer/inner surfaces as $BaTio_3/CoFe_2O_3$ and an aspect ratio of 4 is considered. The rotating cylinder is under unit internal pressure and electric potential while the outer surface is free and grounded. Fig. 1 shows the effect of non-dimensional internal temperature on the thermomagnetoelastic response of the cylinder through the thickness. The nondimensional location parameter ρ is equal to the normalized radial coordinate r by the inner radius of the cylinder. Since orthotropic smart cylinders were reported to fail at a critical hoop stress, the effect of magnetic boundary condition and angular velocity is investigated to reduce the hoop stress in the cylinder.



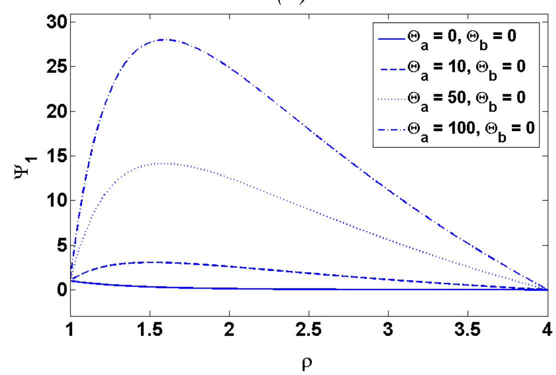

*(a)*




*(b)*




*(c)*




*(d)*

Fig. 1 Effect of the internal thermal boundary condition on the distribution of: *(a)* radial stress, *(b)* hoop stress, *(c)* magnetic potential, and *(d)* magnetic induction.

## References

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