Optimal Vibration Control of Smart Fibre Reinforced Polymer Spherical Shells

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Introduction

Integration of distributed or discrete piezoelectric sensors/actuators to advanced fiber reinforced polymer (FRP) composites using appropriate control technology led to light weight smart structures capable of self-monitoring and self–controlling. Linear quadratic regulator (LQR) approach has been extensively used for active vibration control of such structures with appropriate weighting matrices, which gives optimal control gain by minimizing the performance index. Some of the relevant works in this direction are presented in the following paragraph.

Bhattacharya et al [1] used LQR strategy for vibration suppression of smart FRP spherical shells. Ang et al [2] proposed the use of total weighted energy method to select the weighting matrices. Narayanan and Balamurugan [3] presented finite element modeling of smart laminated structures and applied LQR control scheme to control the displacement. Yang and Soh [4] presented a simultaneous optimization method by minimizing the equivalent total mechanical energy of the system but did not show the actuators voltages.

In, all the published works weighting matrices *[Q]* and *[R]* have been chosen by trial and error even though they decide the optimal gain in LQR control and very few works discussed about actuation voltage while maximizing control performance. The present work thus aims at developing a genetic algorithm (GA) based LQR optimal vibration control methodology of smart structures for determination of optimal gain so that input /actuation voltage is kept within limit. Finite Element (FE) analysis has been used to obtain the coupled electro-mechanical responses of the smart shell structures.

Formulation

Using linear piezoelectric constitutive equations coupling the elastic and electric fields, the overall dynamic finite element equation is

$$\left[M_{uu}\right]\left\{\ddot{d}\right\}+\left[[K_{uu}]-[K_{ua}][K_{aa}]^{-1}[K_{au}]-[K_{us}][K_{ss}]^{-1}[K_{su}]\right]\{d\}=\{F\}-\left[K_{ua}\right]\{\phi_a\} \quad (1)$$

where $\left[M_{uu}\right]$ is the global mass matrix, $\left[K_{uu}\right]$ is the global elastic stiffness matrix, $\left[K_{ua}\right]$ and $\left[K_{us}\right]$ are the global piezoelectric coupling matrices of actuator and sensor patches respectively. $\left[K_{aa}\right]$ and $\left[K_{ss}\right]$ are the global dielectric stiffness matrices of actuator and sensor patches respectively. Eight noded shell finite elements having five degrees of freedom at each node have been used. The decoupled dynamic equations considering modal damping can be written as

$$\left\{\ddot{\eta}_i(t)\right\}+2\xi_{di}\omega_i\left\{\dot{\eta}_i(t)\right\}+\omega_i^2\left\{\eta_i(t)\right\}=\left[\psi\right]^T\{F\}-\left[\psi\right]^T\left[K_{ua}\right]\{\phi_a\} \quad (2)$$

where ξ_{di} is the damping ratio. In state-space form

$\left\{\dot{X}\right\}=\left[A\right]\{X\}+\left[B\right]\{\phi_a\}+\left[\hat{B}\right]\{u_d\}$ (3) and $\left\{\dot{X}\right\}=\begin{Bmatrix}\dot{\eta}\\ \ddot{\eta}\end{Bmatrix}$ and $\{X\}=\begin{Bmatrix}\eta\\ \dot{\eta}\end{Bmatrix}$

LQR optimal control theory has been used to determine the control gains by minimizing a cost function or a performance index given by

Minimize $J=\frac{1}{2}\int_{t_0}^{t_f}(\{y\}^T[Q]\{y\}+\{\phi_a\}^T[R]\{\phi_a\})dt$ (4)

which leads to the optimal gain as $\left[G_c\right]=[R]^{-1}[B]^T[K]$ (5)

Considering output feedback, actuation voltage can be calculated as $\{\phi_a\}=-[G_c]\{y\}$ (6)

By putting [*Q*] and [*R*] matrices as [2]

$[Q]=\begin{bmatrix}\alpha_2[\psi]^T[K][\psi] & [0]\\ [0] & \alpha_1[\psi]^T[M][\psi]\end{bmatrix}$, and $[R]=\gamma[\hat{R}]$ the weighted energy of the system in the quadratic form is

$$\bar{\Pi}=\frac{1}{2}\alpha_1\{\dot{X}\}^T[M]\{\dot{X}\}+\frac{1}{2}\alpha_2\{X\}^T[K]\{X\}+\frac{1}{2}\gamma\{\phi_a\}^T[\hat{R}]\{\phi_a\} \quad (7)$$

where, α_1,α_2 and γ are the coefficients associated with total kinetic energy, strain energy and input energy respectively. These coefficients will take different values in the control algorithm apart from the value of unity to allow for the relative importance of these energy terms. In the present GA based LQR approach, parameters α_1, α_2 and γ in Eq. (7) have been represented by real-valued genes for finding [*Q*] and [*R*] matrices. The ranges of α_1,α_2 and γ are taken as $0<\alpha_1\le 200,\quad 0<\alpha_2\le 200$ and $0<\gamma\le 2$. Parents have been selected through roulette wheel operator and offspring have been created using simulated binary crossover and polynomial mutation operator. Genetic evolution has been continued for large number of generations till the fitness converges.

The search algorithm is

$Maximize\ \xi_d=\frac{1}{\sqrt{\left(1+\frac{4\pi^2}{p^2}\right)}}$ subjected to $\phi_i<\phi_{max}, i=1,\ldots n_a$

where $p=\ln\left(\frac{x_i}{x_{i+1}}\right)$, n_a is the number of actuators and ϕ_{max} refers to the maximum voltage that can be applied on the actuators.

Results and Discussions

A simply supported smart GR/E spherical shell (Fig. 1) under the action of impulse load at the center has been analyzed to study the vibration. Here, $a=b=20\times10^{-3}$m, $R_1=R_2=60\times10^{-3}$m. Properties of graphite/epoxy are $E_1=25E_2, G_{12}=G_{13}=.5E_2, \nu_{12}=0.25, G_{23}=.2E_2, E_2=6.9\,GPa$. Stacking sequence considered is p/[0/+45/-45/90]$_2$/p Here 'p' stands for piezo-patches one for sensing and the other for actuation. Thickness of each ply is 0.25 mm and that of piezo-patch is 0.5 mm. The spherical shell structure has been discretized into 10×10 finite element.

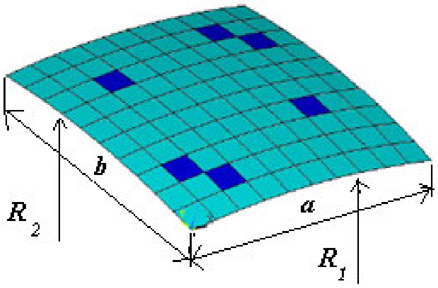

Figure 1. Smart FRP spherical shell

The spherical panel has been subjected to an impulse load of 10 N at the center for duration of 3.05 µs and impulse responses of the panel have been calculated. Optimum actuator/sensor placement obtained by another program has been used as shown in Fig.1 and 1% proportional damping has been considered. Figure 2 shows the LQR and GA-LQR controlled displacement histories obtained from the present code. It has been observed from Fig. 2 that the closed loop-damping ratio obtained is (12.1%) much more in the case of GA-LQR search control scheme than that in the case of simple LQR control scheme (1.94%). This shows that choosing optimal gain through the proposed GA-LQR method maximizes the closed loop damping where as the conventional LQR fails. The maximum actuator voltage variations using LQR and GA-LQR control scheme is also shown in Fig. 3. From these results it could be concluded that the GA-LQR control scheme led to the maximization of closed loop damping ratio keeping the maximum input/actuator voltage within the limit.

Conclusion

A GA based LQR control scheme has been developed in order to maximize closed loop damping ratio within the limit of input/actuator voltage for optimal active vibration control of smart FRP composite shell structures. The proposed GA-LQR control scheme has been observed to find better optimal gain compared to conventional LQR scheme leading to a much higher effective closed loop damping.

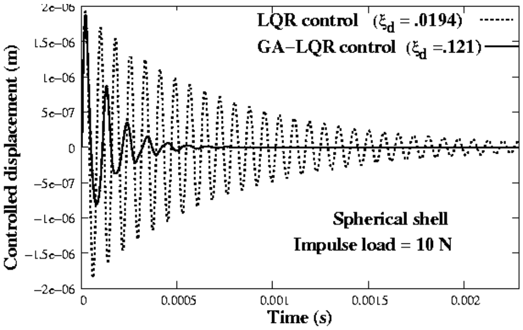

Figure 2. Controlled displacement history

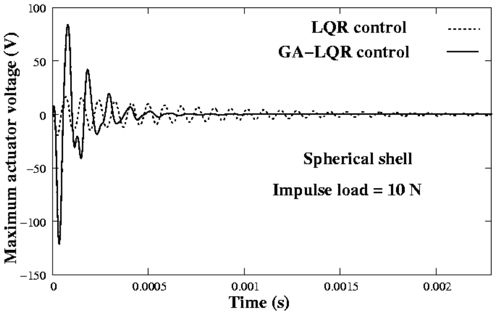

Figure 3. Maximum actuator voltage history

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