INTERNAL RESONANCE OF COMPOSITE SANDWICH PANELS WITH TRANSVERSELY COMPRESSIBLE CORE

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Introduction

A typical sandwich panel is a layered structure consisting of two thin face sheets which are bonded to a thick core layer. This type of structures have rich dynamic behavior due to its composite configuration.

In this article, we investigate internal resonances of composite sandwich panels with transversely compressible core. Geometrically nonlinear sandwich panels can have manifold nonlinear characteristics such as multiple solutions, limit cycles, subharmonic and superharmonic resonances, period-multiplying bifurcations and chaotic motions. In addition to the above mentioned phenomena, sandwich panels can also have the response of combination resonances and modal interactions because of multiple degrees of freedom. In Table 1, the first 13 eigenfrequencies are calculated to compare with the experimental and numerical results by Raville and Ueng[1]. We readily find some interesting combinations among the linear natural frequencies ω_{mn}, e.g., $2\omega_{11} \approx \omega_{21}, \omega_{11} + \omega_{12} \approx \omega_{22}, \omega_{11} + \omega_{21} \approx \omega_{12}$, $\omega_{21} + \omega_{31} \approx \omega_{32}$, $2\omega_{12} \approx \omega_{13}$, $\omega_{12} + \omega_{41} \approx \omega_{33}$, and $\omega_{32} \approx \omega_{41}, \omega_{12} + \omega_{22} \approx \omega_{23}$. Internal resonances may occur if the linear natural frequencies are commensurable or nearly commensurable. Since we have found in this case so many commensurable combinations among the first 13 linear natural frequencies that it is necessary to investigate the internal resonances of composite sandwich panels.

Table 1 Eigenfrequencies [HZ] for a flat sandwich panel.

m	n 1	2	3	4	
1	23.5	71.0	146.5	245.3	Present study
		69.0	152.0	246.0	Literature[1](exp.)
	23.0	71.0	146.0	244.0	Literature[2](num.)
2	45.1	92.1	166.7	264.5	Present study
	45.0	92.0	169.0	262.0	Literature[1](exp.)
	45.0	91.0	165.0	263.0	Literature[2](num.)
3	80.7	126.8	200.1		Present study
	78.0	129.0	199.0		Literature[1](exp.)
	80.0	126.0	195.0		Literature[2](num.)
4	130.0	174.9			Present study
	133.0	177.0			Literature[1](exp.)
	129.0	174.0			Literature[2](num.)

Statement of the problem

Here, a geometrically nonlinear composite sandwich panel with transversely compressible core is investigated. The panel is assumed to be symmetric with respect to the global mid-surface and the thicknesses of the face sheets and core are denoted by t^f and t^c ,respectively. The displacements $v_i(i=\overline{1,3})$ of the structure are expressed as a power series with respect to the transverse coordinate x_3 . For the face sheets, the classical Kirchhoff-Love theory is adopted because of the small thickness. For the core, a higher-order expansion has to be used for the representation of the core displacement filed owing to the large thickness and the transverse compressibility. Considering the displacement continuity requirements, the displacements of the core are obtained in the form[2]

$$v_1^c = u_1^a - \frac{t^f}{2}u_{3,1}^d - \frac{2x_3}{t^c}u_1^d + \frac{t^f}{t^c}x_3u_{3,1}^a + [\frac{4(x_3)^2}{(t^c)^2} - 1]\Omega_1^c$$

$$v_2^c = u_2^a - \frac{t^f}{2}u_{3,2}^d - \frac{2x_3}{t^c}u_2^d + \frac{t^f}{t^c}x_3u_{3,2}^a + [\frac{4(x_3)^2}{(t^c)^2} - 1]\Omega_2^c$$

$$v_3^c = u_3^a - \frac{2x_3}{t^c}u_3^d$$

where $\Omega_\alpha^c(\alpha = 1,2)$ are additional displacement functions describing the warping of the core. The Green-Lagrange strain tensor is used to describe the strain of the sandwich panel. The nonlinear terms related to the in-plane displacements are discarded according to the von Karman's theory. For simplicity, the constitutive relations for the face sheet and the core are both assumed to be orthotropic.

Equations of motion

On the basis of the assumption we made above, the equations of motion and the boundary conditions can be derived from Hamilton's principle. Then, the equations of motion can be represented as follows:

$$\delta u_1^a : N_{1,1}^a + N_{12,2}^a = 0$$

$$\delta u_2^a : N_{2,2}^a + N_{12,1}^a = 0$$

$$\delta u_1^d : N_{1,1}^d + N_{12,2}^d = -\frac{N_{13}^c}{t^c}$$

$$\delta u_2^d : N_{2,2}^d + N_{12,1}^d = -\frac{N_{23}^c}{t^c}$$

$$\delta u_3^a : -(m_0^f + \frac{m_0^c}{2})\ddot{u}_3^a + N_1^a u_{3,11}^a + 2N_{12}^a u_{3,12}^a + N_2^a u_{3,22}^a + 2M_{12,12}^a + M_{1,11}^a + M_{2,22}^a + N_1^d u_{3,11}^d + 2N_{12}^d u_{3,12}^d + N_2^d u_{3,22}^d - \frac{2N_{13}^c}{t^c} u_{3,1}^d - \frac{2N_{23}^c}{t^c} u_{3,2}^d + \frac{1}{t^c}(N_{13,1}^c + N_{23,2}^c) * (\frac{t^c + t^f}{2} - u_3^d) + q_3^a = 0$$

$$\delta u_3^d : -(m_0^f + \frac{m_0^c}{2})\ddot{u}_3^d + N_1^d u_{3,11}^a + 2N_{12}^d u_{3,12}^a + N_2^d u_{3,22}^a + 2M_{12,12}^d + M_{1,11}^d + M_{2,22}^d + N_1^a u_{3,11}^d + 2N_{12}^a u_{3,12}^d + N_2^a u_{3,22}^d + \frac{2}{(t^c)^2} N_3^c (\frac{t^c}{2} - u_3^d) + q_3^d = 0$$

Solution

The first two equations are simplified by introducing an Airy stress function Φ. Then we introduce some basic nondimensional terms:

$$U_1^d \equiv \frac{u_1^d}{l_1}, U_2^d \equiv \frac{u_2^d}{l_2}, U_3^a \equiv \frac{u_3^a}{t^c + 2t^f}, U_3^d \equiv \frac{u_3^d}{t^c + 2t^f},$$

$$X_1 \equiv \frac{x_1}{l_1}, X_2 \equiv \frac{x_2}{l_2}, X_3 \equiv \frac{x_3}{t^c + 2t^f}, \tau \equiv \omega^* t, \varphi \equiv \frac{\Phi}{D}$$

We then use the method of multiple scales[3] to expand the dimensionless equations. According to this method, new independent variables are introduced as follows:

$$U_i = \varepsilon U_{i1} + \varepsilon^2 U_{i2} + \varepsilon^3 U_{i3} + \cdots$$

$$T_0 = \tau, T_1 = \varepsilon\tau, T_2 = \varepsilon^2\tau, \cdots, T_n = \varepsilon^n\tau$$

A set of equations for each power of ε are obtained by substituting the new variables into the dimensionless equations. We then introduce the following solution form:

$$U_{11}^d = A_{mn} W_{mn}^a \cos(m\pi X_1)\sin(n\pi X_2) \quad U_{21}^d = B_{mn} W_{mn}^a \sin(m\pi X_1)\cos(n\pi X_2)$$

$$U_{31}^a = W_{mn}^a \sin(m\pi X_1)\sin(n\pi X_2) \quad U_{31}^d = W_{pq}^d \sin(p\pi X_1)\sin(q\pi X_2)$$

$$U_{12}^d = V_{12}^d \cos(m\pi X_1)\sin(n\pi X_2) \quad U_{22}^d = V_{22}^d \sin(m\pi X_1)\cos(n\pi X_2)$$

$$U_{32}^a = V_{32}^a \sin(m\pi X_1)\sin(n\pi X_2) \quad U_{32}^d = V_{32}^d \sin(p\pi X_1)\sin(q\pi X_2)$$

where m and n are the modal wave numbers with respect to the $x_1 - and\ x_2 -$ directions of the global deformation whereas the p and q are of the local deformation. At last, the set of equations can be expressed as follows:

$$0(\varepsilon): \quad M_1^a \ddot{W}_{mn}^a + K_1^a W_{mn}^a = 0$$
$$M_1^d \ddot{W}_{pq}^d + K_1^d W_{pq}^d = 0$$

$$0(\varepsilon^2): \quad M_2^a \ddot{V}_{32}^a + K_2^a V_{32}^a = L_2^a W_{mn}^a W_{pq}^d$$
$$M_2^d \ddot{V}_{32}^d + K_2^d V_{32}^d = Q_2^d (W_{mn}^a)^2 + R_2^d (W_{pq}^d)^2$$

$$0(\varepsilon^3): \quad M_3^a \ddot{V}_{33}^a + K_3^a V_{33}^a = F_1[W_{mn}^a V_{32}^d, W_{pq}^d V_{32}^a, W_{mn}^a (W_{pq}^d)^2, (W_{mn}^a)^3]$$
$$M_3^d \ddot{V}_{33}^d + K_3^d V_{33}^d = F_2[W_{mn}^a V_{32}^a, W_{pq}^d V_{32}^d, (W_{mn}^a)^2 W_{pq}^d, (W_{pq}^d)^3]$$

The equations with respect to ε are solved in the form

$$W_{mn}^a = Z_1 e^{i\omega_{mn}^a T_0} + cc \quad W_{pq}^d = Z_2 e^{i\omega_{pq}^d T_0} + cc$$

where $\omega_{mn}^a\ and\ \omega_{pq}^d$ (ω_{mn}^a are calculated in Table 1)are the functions of m, n and p, q, respectively. Then , the solution of the equations with respect to ε^2 are obtained by substituting the solution above as follows:

$$V_{32}^a = V_2 e^{i(\omega_{mn}^a + \omega_{pq}^d)T_0} + V_3 e^{i(\omega_{mn}^a - \omega_{pq}^d)T_0} + cc$$

$$V_{32}^d = V_5 e^{i2\omega_{mn}^a T_0} + V_6 e^{i2\omega_{pq}^d T_0} + cc + r$$

We substitute the solution all of above into the equations with respect to ε^3. The nonlinear coupling part (the linear part will have the same form with the first two equations)are obtained in the form:

$$F_1[e^{\pm i\omega_{mn}^a T_0}, e^{\pm i3\omega_{mn}^a T_0}, e^{\pm i(\omega_{mn}^a + 2\omega_{pq}^d)T_0}, e^{\pm i(\omega_{mn}^a - 2\omega_{pq}^d)T_0}]$$

$$F_2[e^{\pm i\omega_{pq}^d T_0}, e^{\pm i3\omega_{pq}^d T_0}, e^{\pm i(2\omega_{mn}^a + \omega_{pq}^d)T_0}, e^{\pm i(2\omega_{mn}^a - \omega_{pq}^d)T_0}]$$

where $Z_1\ and\ Z_2$ are unknown all along. The coefficients of $e^{i\omega_{mn}^a T_0}\ and\ e^{i\omega_{pq}^d T_0}$ must vanish according to the solvability condition which would leads to a set of coupled, first-order differential equations for $Z_1\ and\ Z_2$. The 1:1 internal resonance is obtained when $\omega_{mn}^a = \omega_{pq}^d$ by joining the expression of external loads $q_3^a = \hat{q}e^{i\Omega T_0} (\Omega T_0 = \omega_{mn}^a T_0 + \sigma T_2)$ where σ is a very small parameter.

Conclusions

The 1:1 internal resonance is obtained by using just a single-mode expansion. If we use multi-mode expansion, more possible combinations of internal resonances can be found, which is what we are investigating.

References

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3. A.H. Nayfeh, D.T. Mook, D.W. Lobitz, Numerical-perturbation method for the nonlinear analysis of structural vibrations, *AIAA J.*,12:1222-1228.